Proposed Washington State K–8 Mathematics Standards

April 28, 2008

**Office of the Superintendent
of Public Instruction**

Kindergarten
K.1. Core Content: *Whole numbers* *(Numbers, Operations)*

Students begin to develop basic notions of numbers and use numbers to think about objects and the world around them. They practice counting objects in sets, and they think about how numbers are ordered by showing the numbers on the number line. As they put together and take apart simple numbers, students lay the groundwork for learning how to add and subtract. Understanding numbers is perhaps the most central idea in all of mathematics, and if students build and maintain a strong foundation of number sense and number skills, they will be able to succeed with increasingly sophisticated numerical knowledge and skills from year to year.

Performance Expectations	*Explanatory Comments and Examples*
Students are expected to:	
K.1.A Rote count by ones forward from 1 to 100 and backward from any number in the range of 10 to 1.	
K.1.B Read aloud numerals from 0 to 31.	Shown numeral cards in random order from 0 to 31, students respond with the correct name of the numerals. Students also demonstrate that they can distinguish 12 from 21 and 13 from 31—a common challenge for kindergartners. The choice of 31 corresponds to the common use of calendar activities in kindergarten.
K.1.C Fluently compose and decompose numbers to 5.	Students should be able to state that 5 is made up of 4 and 1, 3 and 2, 2 and 3, or 1 and 4. They should understand that if I have 3, I need 2 more to make 5, or that if I have 4, I need only 1 more to make 5. Students should also be able to recognize the number of missing objects without counting. The words *compose* and *decompose* are used to describe actions that young students learn as they acquire knowledge of small numbers by putting them together and taking them apart. This understanding is a bridge between counting and knowing number combinations. It is how instant recognition of small numbers develops and leads naturally to later understanding of fact families. Example: • Here are 5 counters. I will hide some. If you see 2, how many am I hiding?
K.1.D Order numerals from 1 to 10.	The student takes numeral cards (1 to 10) that have been shuffled and puts them in correct ascending order.

Performance Expectations	Explanatory Comments and Examples
Students are expected to:	

K.1.E   Count objects in a set of up to 20, and count out a specific number of up to 20 objects from a larger set.

K.1.F   Compare two sets of up to 10 objects each and say whether the number of objects in one set is equal to, greater than, or less than the number of objects in the other set.

K.1.G   Locate numbers from 1 to 31 on the number line.

Students should be able to do this without having to start counting at 1.

K.1.H   Describe a number from 1 to 9 using 5 as a benchmark number.

Students should make observations such as "7 is 2 more than 5" or "4 is 1 less than 5." This is helpful for mental math and lays the groundwork for using 10 as a benchmark number in later work with base-ten numbers and operations.

Kindergarten
K.2. Core Content: Patterns and operations   *(Operations, Algebra)*

Students learn what it means to add and subtract by joining and separating sets of objects. Working with patterns helps them strengthen this understanding of addition and subtraction and moves them toward the important development of algebraic thinking. Students study simple repetitive patterns in preparation for increasingly sophisticated patterns that can be represented with algebraic expressions in later grades.

Performance Expectations	*Explanatory Comments and Examples*
Students are expected to:	
K.2.A Copy, extend, describe, and create simple repetitive patterns.	Students can complete these activities with specified patterns of the type AB, AAB, AABB, ABC, etc.

Examples:

- Make a type AB pattern of squares and circles with one square, one circle, one square, one circle, etc.

- Here is a type AAB pattern using colored cubes: red, red, blue, red, red, blue, red, red. What comes next?

- A shape is missing in the type AB pattern below. What is it?

K.2.B Translate a pattern among sounds, symbols, movements, and physical objects.

Red, *red*, *yellow*, *red*, *red*, *yellow* could translate to *clap*, *clap*, *snap*, *clap*, *clap*, *snap*.

Students should be able to translate patterns among all of these representations. However, when they have demonstrated they can do this, they need not use all representations every time.

K.2.C Model addition by joining sets of objects that have 10 or fewer total objects when joined and model subtraction by separating a set of 10 or fewer objects.

Seeing two sets of counters or other objects, the student determines the correct combined total. The student may count the total number of objects in the set or use some other strategy in order to arrive at the sum. The student establishes the total number of counters or objects in a set; then, after some have been removed, the student figures out how many are left.

Examples:

- Get 4 counting chips. Now get 3 counting chips. How many counting chips are there altogether?

- Get 8 counting chips. Take 2 away. How many are left?

K.2.D   Describe a situation that involves the actions of joining (addition) or separating (subtraction) using words, pictures, objects, or numbers.

Students can be asked to tell an addition story or a subtraction story.

Kindergarten

K.3. Core Content: *Objects and their locations*   *(Geometry/Measurement)*

Students develop basic ideas related to geometry as they name simple two- and three-dimensional figures and find these shapes around them. They expand their understanding of space and location by describing where people and objects are. Students sort and match shapes as they begin to develop classification skills that serve them well in both mathematics and reading—matching numbers to sets, shapes to names, patterns to rules, letters to sounds, and so on.

Performance Expectations	*Explanatory Comments and Examples*
Students are expected to:	
K.3.A Identify, name, and describe circles, triangles, rectangles, squares (as special rectangles), cubes, and spheres.	Students should be encouraged to talk about the characteristics (e.g., round, four-cornered) of the various shapes and to identify these shapes in a variety of contexts regardless of their location, size, and orientation. Having students identify these shapes on the playground, in the classroom, and on clothing develops their ability to generalize the characteristics of each shape.
K.3.B Sort shapes using a sorting rule and explain the sorting rule.	Students could sort shapes according to attributes such as the shape, size, or the number of sides, and explain the sorting rule. Given a selection of shapes, students may be asked to sort them into two piles and then describe the sorting rule. After sorting, a student could say, "I put all the round ones here and all the others there."
K.3.C Describe the location of one object relative to another object using words such as *in*, *out*, *over*, *under*, *above*, *below*, *between*, *next to*, *behind*, and *in front of*.	Examples: • Put this pencil under the paper. • I am between José and Katy.

Kindergarten

K.4. Additional Key Content   *(Geometry/Measurement)*

Students informally develop early measurement concepts. This is an important precursor to Core Content on measurement in later grades, when students measure objects with tools. Solving measurement problems connects directly to the student's world and is a basic component of learning mathematics.

Performance Expectations	*Explanatory Comments and Examples*
Students are expected to:	
K.4.A Make direct comparisons using measurable attributes such as length, weight, and capacity.	Students should use language such as *longer than*, *shorter than*, *taller than*, *heavier than*, *lighter than*, *holds more than,* or *holds less than*.

Kindergarten

K.5. Core Processes: *Reasoning, problem solving, and communication*

Students begin to build the understanding that doing mathematics involves solving problems and discussing how they solved them. Problems at this level emphasize counting and activities that lead to emerging ideas about addition and subtraction. Students begin to develop their mathematical communication skills as they participate in mathematical discussions involving questions like "How did you get that?" and "Why is that true?"

Performance Expectations	*Explanatory Comments and Examples*
Students are expected to:	

K.5.A	Identify the question(s) asked in a problem.	Descriptions of solution processes and explanations can include numbers, words (including mathematical language), pictures, or physical objects. Students should be able to use all of these representations as needed. For a particular solution, students should be able to explain or show their work using at least one of these representations and verify that their answer is reasonable.
K.5.B	Identify the given information that can be used to solve a problem.	
K.5.C	Recognize when additional information is required to solve a problem.	
K.5.D	Select from a variety of problem-solving strategies and use one or more strategies to solve a problem.	
K.5.E	Answer the question(s) asked in a problem.	Examples:
K.5.F	Describe how a problem was solved.	• Grandma went to visit her two grandchildren and discovered that the gloves they were each wearing had holes in every finger, even the thumbs. She will fix their gloves. How many glove fingers (including thumbs) need to be fixed?
K.5.G	Determine whether a solution to a problem is reasonable.	• Students are given drinking straws or coffee stirrers cut to a variety of different lengths: 6″, 5″, 4″, 3″, and 2″. They are to figure out which sets of three lengths, when joined at their ends, will form triangles and which sets of three will not.

Grade 1

1.1. Core Content: *Whole number relationships* *(Numbers, Operations)*

Students continue to work with whole numbers to quantify objects. They consider how numbers relate to one another. As they expand the set of numbers they work with, students start to develop critical concepts of ones and tens that introduce them to place value in our base ten number system. An understanding of how ones and tens relate to each other allows students to begin adding and subtracting two-digit numbers, where thinking of ten ones as one ten and vice versa is routine. Some students will be ready to work with numbers larger than those identified in the *Expectations* and should be given every opportunity to do so.

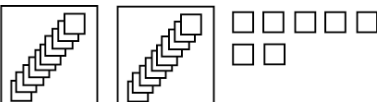| Performance Expectations	Explanatory Comments and Examples
Students are expected to:	
1.1.A Count by ones forward and backward from 1 to 120, starting at any number, and count by twos, fives, and tens to 100.	Research suggests that when students count past 100, they often make errors such as "99, 100, 200" and "109, 110, 120." However, once a student counts to 120 consistently, it is highly improbable that additional counting errors will be made.
	Example:
	• Start at 113. Count backward. I'll tell you when to stop. [Stop when the student has counted backward ten numbers.]
1.1.B Name the number that is one less or one more than any number given verbally up to 120.	
1.1.C Read aloud numerals from 0 to 1,000.	The patterns in the base ten number system become clearer to students when they count in the hundreds. Therefore, learning the names of three-digit numbers supports the learning of more difficult two-digit numbers (such as numbers in the teens and numbers ending in 0 or 1).
1.1.D Order objects or events using ordinal numbers.	Students use ordinal numbers to describe positions through the twentieth.
	Example:
	• John is fourth in line.
1.1.E Write, compare, and order numbers to 120.	Students arrange numbers in lists or talk about the relationships among numbers using the words *equal to*, *greater than*, *less than*, *greatest*, and *least*.
	Example:
	• Write the numbers 27, 2, 111, and 35 from least to greatest.
	Students might also describe which of two numbers is closer to a given number. This is part of developing an understanding of the relative value of numbers.

1.1.F Fluently compose and decompose numbers to 10.

Students put together and take apart whole numbers as a precursor to addition and subtraction.
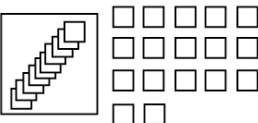
Examples:

- Ten is 2 + 5 + 1 + 1 + 1.

- Eight is five and three.

- Here are twelve coins. I will hide some. If you see three, how many am I hiding? [This example demonstrates how students might be encouraged to go beyond the expectation.]

1.1.G Group numbers into tens and ones in more than one way.

Students demonstrate that the value of a number remains the same regardless of how it is grouped. Grouping of numbers lays a foundation for future work with addition and subtraction of two-digit numbers, where renaming may be necessary.
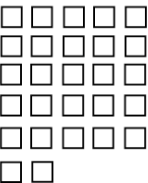
For example, twenty-seven objects can be grouped as 2 tens and 7 ones, regrouped as 1 ten and 17 ones, and regrouped again as 27 ones. The total (27) remains constant.

$$27 = 10 + 10 + 7$$

$$27 = 10 + 17$$

1.1.H Group and count objects by tens, fives, and twos.

Given 23 objects, the student will count them by tens as 10, 20, 21, 22, 23; by fives as 5, 10, 15, 20, 21, 22, 23; and by twos as 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 23.

Performance Expectations	**Explanatory Comments and Examples**
Students are expected to:	

1.1.I Classify a number as odd or even and demonstrate that it is odd or even.

Students use words, objects, or pictures to demonstrate that a given number is odd or even.

Example:

- 13 is odd because 13 counters cannot be regrouped into two equal piles.

- 20 is even because every counter in this set of 20 counters can be paired with another counter in the set.

Grade 1

1.2. Core Content: *Addition and subtraction*   *(Operations, Algebra)*

Students learn how to add and subtract, when to add and subtract, and how addition and subtraction relate to each other. Understanding that addition and subtraction undo each other is an important part of learning to use these operations efficiently and accurately. Students notice patterns involving addition and subtraction, and they work with other types of patterns as they learn to make generalizations about what they observe.

Performance Expectations	*Explanatory Comments and Examples*
Students are expected to:	

1.2.A Connect physical and pictorial representations to addition and subtraction equations.

The intention of the standard is for students to understand that mathematical equations represent situations. Simple student responses are adequate.

Combining a set of 3 objects and a set of 5 objects to get a set of 8 objects can be represented by the equation $3 + 5 = 8$. The equation $2 + 6 = 8$ could be represented by drawing a set of 2 cats and a set of 6 cats making a set of 8 cats. The equation $9 - 5 = 4$ could be represented by taking 5 objects away from a set of 9 objects.

1.2.B Use the equal sign (=) and the word *equals* to indicate that two expressions are equivalent.
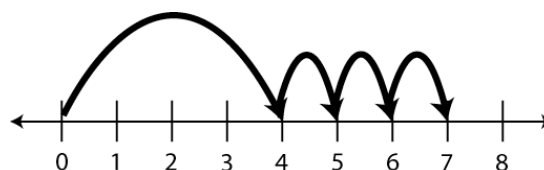
Students need to understand that *equality* means *is the same as*. This idea is critical if students are to avoid common pitfalls in later work with numbers and operations, where they may otherwise fall into habits of thinking that the answer always follows the equal sign.
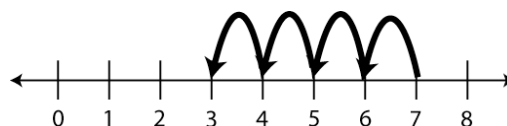
Examples:

- $7 = 8 - 1$

- $5 + 3$ equals $10 - 2$

1.2.C Represent addition and subtraction on the number line.

Examples:

- $4 + 3 = 7$

- $7 - 4 = 3$

Performance Expectations	Explanatory Comments and Examples
Students are expected to:	

1.2.D   Demonstrate the inverse relationship between addition and subtraction by undoing an addition problem with subtraction and vice versa.

The relationship between addition and subtraction is an important part of developing algebraic thinking. Students can demonstrate this relationship using physical models, diagrams, numbers, or acting-out situations.

Examples:

- $3 + 5 = 8$, so $8 - 3 = 5$

- Annie had ten marbles, but she lost three. How many marbles does she have? Joe found her marbles and gave them back to her. Now how many does she have?

1.2.E   Add three or more one-digit numbers using the commutative and associative properties of addition.

Examples:

- $3 + 5 + 5 = 3 + 10$
  (Associativity allows us to add the last two addends first.)

- $(5 + 3) + 5 = 5 + (5 + 3) = (5 + 5) + 3 = 13$
  (Commutativity and associativity allow us to reorder addends.)

This concept can be extended to address a problem like $3 + \triangle + 2 = 9$, which can be rewritten as $5 + \triangle = 9$.

1.2.F   Apply and explain strategies to compute addition facts and related subtraction facts for sums to 18.

Strategies for addition include counting on, but students should be able to move beyond counting on to use other strategies, such as making 10, using doubles or near doubles, etc.

Subtraction strategies include counting back, relating the problem to addition, etc.

1.2.G   Quickly recall addition facts and related subtraction facts for sums equal to 10.

Adding and subtracting zero are included.

1.2.H   Solve and create word problems that match addition or subtraction equations.

Students should be able to represent addition and subtraction sentences with an appropriate situation, using objects, pictures, or words. This standard is about helping students connect symbolic representations to situations. While some students may create word problems that are detailed or lengthy, this is not necessary to meet the expectation. Just as we want students to be able to translate 5 boys and 3 girls sitting at a table into $5 + 3 = 8$, we want students to look at an expression like $7 - 4 = 3$ and connect it to a situation or problem using objects, pictures, or words.

Example:

- For the equation 7 + ? = 10, a possible story might be:

  Jeff had 7 marbles in his pocket and some marbles in his drawer. He had 10 marbles altogether. How many marbles did he have in his drawer? Use pictures, words, or objects to show your answer.

1.2.I   Recognize, extend, and create number patterns.

Example:

- Extend the simple addition patterns below and tell how you decided what numbers come next:

  1, 3, 5, 7, . . .

  2, 4, 6, 8, 10, . . .

  50, 45, 40, 35, 30, . . .

Grade 1

1.3. Core Content: Geometric attributes   *(Geometry/Measurement)*

Students expand their knowledge of two- and three-dimensional geometric figures by sorting, comparing, and contrasting them according to their characteristics. They learn important mathematical vocabulary used to name the figures. Students work with composite shapes made out of basic two-dimensional figures as they continue to develop their spatial sense of shapes, objects, and the world around them.

Performance Expectations	*Explanatory Comments and Examples*
Students are expected to:	
1.3.A Compare and sort a variety of two- and three-dimensional figures according to their geometric attributes.	The student may sort a collection of two-dimensional figures into those that have a particular attribute (e.g., those that have straight sides) and those that do not.
1.3.B Identify and name two-dimensional figures, including those in real-world contexts, regardless of size or orientation.	Figures should include circles, triangles, rectangles, squares (as special rectangles), rhombi, hexagons, and trapezoids.
	Contextual examples could include classroom clocks, flags, desktops, wall or ceiling tiles, etc. Triangles should appear in many positions and orientations and should not all be equilateral or isosceles.
1.3.C Combine known shapes to create shapes and divide known shapes into other shapes.	Students could be asked to trace objects or use a drawing program to show different ways that a rectangle can be divided into three triangles. They can also use pattern blocks or plastic shapes to make new shapes. The teacher can give students cutouts of shapes and ask students to combine them to make a particular shape.
	Example:
	• What shapes can be made from a rectangle and a triangle? Draw a picture to show your answers.

Grade 1

1.4. Core Content: Concepts of measurement   *(Geometry/Measurement)*

Students start to learn about measurement by measuring length. They begin to understand what it means to measure something, and they develop their measuring skills using everyday objects. As they focus on length, they come to understand that units of measure must be equal in size and learn that standard-sized units exist. They develop a sense of the approximate size of those standard units (like inches or centimeters) and begin using them to measure different objects. Students learn that when a unit is small, it takes more of the unit to measure an item than it does when the units are larger, and they relate and compare measurements of objects using units of different sizes. Over time they apply these same concepts of linear measurement to other attributes such as weight and capacity. As students practice using measurement tools to measure objects, they reinforce their numerical skills and continue to develop their sense of space and shapes.

Performance Expectations	Explanatory Comments and Examples
Students are expected to:	
1.4.A Recognize that objects used to measure an attribute (length, weight, capacity) must be consistent in size.	Marbles can be suitable objects for young children to use to measure weight, provided that all the marbles are the same weight. Paper clips are appropriate for measuring length as long as the paper clips are all the same length.
1.4.B Use a variety of non-standard units to measure length.	Use craft sticks, toothpicks, coffee stirrers, etc., to measure length.
1.4.C Compare lengths using the transitive property.	Example: • If Jon is taller than Jacob, and Jacob is taller than Luisa, then Jon is taller than Luisa.
1.4.D Use non-standard units to compare objects according to their capacities or weights.	Examples can include using filled paper cups to measure capacity or a balance with marbles or cubes to measure weight.
1.4.E Describe the connection between the size of the measurement unit and the number of units needed to measure something.	Examples: • It takes more toothpicks than craft sticks to measure the width of my desk. The longer the unit, the fewer I need. • It takes fewer marbles than cubes to balance my object. The lighter the unit, the more I need. • It takes more little medicine cups filled with water than larger paper cups filled with water to fill my jar. The less my unit holds, the more I need.

Performance Expectations

Students are expected to:

1.4.F   Name the days of the week and the months of the year, and use a calendar to determine a day or month.

Explanatory Comments and Examples

Examples:

- Name the days of the week in order.

- Name the months of the year in order.

- How many days until your birthday?

- What month comes next?

- What day was it yesterday?

Grade 1

1.5. Additional Key Content (Data/Statistics/Probability)

Students are introduced to early ideas of statistics by collecting and visually representing data. These ideas reinforce their understanding of the Core Content areas related to whole numbers and addition and subtraction as students ask and answer questions about the data. As they move through the grades, students will continue to apply what they learn about data, making mathematics relevant and connecting numbers to applied situations.

Performance Expectations	Explanatory Comments and Examples
Students are expected to:	
1.5.A Represent data using tallies, tables, picture graphs, and bar-type graphs.	In a picture graph, a single picture represents a single object. Pictographs, where a symbol represents more than one unit, are introduced in grade three when multiplication is developed.
	Students are expected to be familiar with all representations, but they need not use them all in every situation.
1.5.B Ask and answer comparison questions about data.	Students develop questions that can be answered using information from their graphs. For example, students could look at tallies showing the number of pockets on pants for each student today.

Andy   卌 |
Sara ||||
Chris   卌

They might ask questions such as:

— Who has the most pockets?

— Who has the fewest pockets?

— How many more pockets does Andy have than Chris?

Grade 1
1.6. Core Processes: Reasoning, problem solving, and communication

Students further develop the concept that doing mathematics involves solving problems and discussing what they did to solve them. Problems in first grade emphasize addition, subtraction, and solidifying number concepts, and sometimes include precursors to multiplication. Students continue to develop their mathematical communication skills as they participate in mathematical discussions involving questions like "How did you get that?"; "Why did you do that?"; and "How do you know that?" Students begin to build their mathematical vocabulary as they use correct mathematical language appropriate to first grade.

Performance Expectations	*Explanatory Comments and Examples*
Students are expected to:	

Performance Expectations

Students are expected to:

1.6.A   Identify the question(s) asked in a problem.

1.6.B   Identify the given information that can be used to solve a problem.

1.6.C   Recognize when additional information is required to solve a problem.

1.6.D   Select from a variety of problem-solving strategies and use one or more strategies to solve a problem.

1.6.E   Answer the question(s) asked in a problem.

1.6.F   Identify the answer(s) to the question(s) in a problem.

1.6.G   Describe how a problem was solved.

1.6.H   Determine whether a solution to a problem is reasonable.

Explanatory Comments and Examples

Descriptions of solution processes and explanations can include numbers, words (including mathematical language), pictures, or physical objects. Students should be able to use all of these representations as needed. For a particular solution, students should be able to explain or show their work using at least one of these representations and verify that their answer is reasonable.

Examples:

- Think about how many feet a person has. How many feet does a cat have? How many feet does a snail have? How about a fish or a snake?

  There are ten feet living in my house. Who could be living in my house?

  Come up with a variety of ways you can have a total of ten feet living in your house. Use pictures, words, or numbers to show how you got your answer.

- You are in charge of setting up a dining room with exactly twenty places for people to sit. You can use any number and combination of different-shaped tables. A hexagon-shaped table seats six people. A triangle-shaped table seats three people. A square-shaped table seats four people.

  Draw a picture showing which tables and how many of each you could set up so that twenty people have a place to sit. Is there more than one way to do this? How many ways can you find?

Grade 2

2.1. Core Content: *Place value and the base ten system* *(Numbers)*

Students refine their understanding of the base ten number system and use place value concepts of ones, tens, and hundreds to understand number relationships. They become fluent in writing and renaming numbers in a variety of ways. This fluency, combined with the understanding of place value, is a strong foundation for learning how to add and subtract two-digit numbers.

Performance Expectations	Explanatory Comments and Examples
Students are expected to:	
2.1.A Count by tens or hundreds forward and backward from 1 to 1,000, starting at any number.	Example: • Count forward by tens out loud starting at 32.
2.1.B Connect place value models with their numerical equivalents to 1,000.	Understanding the relative value of numbers using place value is an important element of our base ten number system. Students should be familiar with representing numbers using words, pictures (including those involving grid paper), or physical objects such as base ten blocks. Money can also be an appropriate model.
2.1.C Identify the ones, tens, and hundreds place in a number and the digits occupying them.	Examples: • 4 is located in what place in the number 834? • What digit is in the hundreds place in 245?
2.1.D Write three-digit numbers in expanded form.	Examples: • 573 = 500 + 70 + 3 • 600 + 30 + 7 = 637
2.1.E Group three-digit numbers into hundreds, tens, and ones in more than one way.	Students should become fluent in naming and renaming numbers based on number sense and their understanding of place value. Examples: • In the number 647, there are 6 hundreds, there are 64 tens, and there are 647 ones. • There are 20 tens in 200 and 10 hundreds in 1,000. • There are 23 tens in 230. • 3 hundreds + 19 tens + 3 ones describes the same number as 4 hundreds + 8 tens + 13 ones.
2.1.F Compare and order numbers from 0 to 1,000.	Students use the words *equal to*, *greater than*, *less than*, *greatest*, or *least* and the symbols =, <, and >.

Grade 2

2.2. Core Content: *Addition and subtraction*   *(Operations, Geometry/Measurement, Algebra)*

Students focus on what it means to add and subtract as they become fluent with single-digit addition and subtraction facts and develop addition and subtraction procedures for two-digit numbers. Students make sense of these procedures by building on what they know about place value and number relationships and by putting together or taking apart sets of objects. This is students' first time to deal formally with step-by-step procedures (algorithms)—an important component of mathematics where a generalizable technique can be used in many similar situations. Students begin to use estimation to determine if their answers are reasonable.

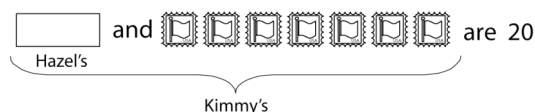| Performance Expectations	Explanatory Comments and Examples
Students are expected to:	
2.2.A Quickly recall basic addition facts and related subtraction facts for sums through 20.	
2.2.B Solve addition and subtraction word problems that involve joining, separating, and comparing and verify the solution.	Problems should include those involving take-away situations, missing addends, and comparisons.
	The intent of this expectation is for students to show their work, explain their thinking, and verify that the answer to the problem is reasonable in terms of the original context and the mathematics used to solve the problem. Verifications can include the use of numbers, words, pictures, or physical objects.
	Example:
	• Hazel and Kimmy each have stamp collections. Kimmy's collection has 7 more stamps than Hazel's. Kimmy has a total of 20 stamps. How many stamps are in Hazel's collection? Explain your answer.
	[Students may verify their work orally, with pictures, or in writing. For instance, students might give the equation below or might use the picture.]
	$$20 - 7 = 13$$
2.2.C Add and subtract two-digit numbers efficiently and accurately using a procedure that works with all two-digit numbers and explain why the procedure works.	Students should be able to connect the numerical procedures with other representations, such as words, pictures, or physical objects.
	This is students' first exposure to mathematical algorithms. It sets the stage for all future work with computational procedures.
	The standard algorithms for addition and subtraction are formalized in grade three.

Performance Expectations	**Explanatory Comments and Examples**
Students are expected to:	

2.2.D   Add and subtract two-digit numbers mentally and explain the strategies used.

Examples of strategies include

- Combining tens and ones:
  68 + 37 = 90 + 15 = 105

- Compensating: 68 + 37 = 65 + 40 = 105

- Incremental: 68 + 37 = 68 + 30 + 7 = 105

2.2.E   Estimate sums and differences.

Example:

- Students might estimate that 198 + 29 is a little less than 230.

2.2.F   Create and state a rule for patterns that can be generated by addition and extend the pattern.

Examples:

- 2, 5, 8, 11, 14, 17, . . .

- Look at the pattern of squares below. Draw a picture that shows what the next set of squares might look like and explain why your answer makes sense.

A   B   C

2.2.G   Solve equations in which the unknown number appears in a variety of positions.
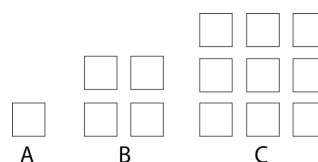
Students need this kind of experience with equivalence to accompany their first work with addition and subtraction. Flexible use of equivalence and missing numbers sets the stage for later work when solving equations in which the variable is in different positions.

Examples:

- $8 + 3 = \square + 5$

- $10 - 7 = 2 + \square$

- $\square = 9 + 4 + 2$

2.2.H   Name each standard U.S. coin, write its value using the $ sign and the ¢ sign, and name combinations of other coins with the same total value.

Students should be expected to express, for example, the value of a quarter as twenty-five cents, $0.25, and 25¢, and they should be able to give other combinations of coins whose value is 25¢. This is a precursor to decimal notation.

2.2.I   Determine the value of a collection of coins totaling less than $1.00.

Grade 2

2.3. Core Content: *Measurement*   *(Geometry/Measurement)*

Students understand the process of measuring length and progress from measuring length with objects such as toothpicks or craft sticks to the more practical skill of measuring length with standard units and tools such as rulers, tape measures, or meter sticks. As students are well acquainted with two-digit numbers by this point, they tell time on different types of clocks.

Performance Expectations	Explanatory Comments and Examples
Students are expected to:	
2.3.A Identify objects that represent or approximate standard units and use them to measure length.	At this level, students no longer rely on non-standard units. Students find and use approximations for standard length units, like a paper clip whose length is about an inch, or the width of a particular student's thumbnail that might be about a centimeter. They might also use commonly available classroom objects like inch tiles or centimeter cubes.
2.3.B Estimate length using metric and U.S. customary units.	Students could make observations such as, "The ceiling of the classroom is about 8 feet high."
2.3.C Measure length to the nearest whole unit in both metric and U.S. customary units.	Standard tools may include rulers, yardsticks, meter sticks, or centimeter/inch measuring tapes. Students should measure some objects that are longer than the measurement tool being used.
2.3.D Describe the relative size among minutes, hours, days, weeks, months, and years.	Students should be able to describe relative sizes using statements like, "Since a minute is less than an hour, there are more minutes than hours in one day."
2.3.E Use both analog and digital clocks to tell time to the minute.	

Grade 2

2.4. Additional Key Content (Numbers, Operations, Geometry/Measurement, Data/Statistics/Probability)*

Students make predictions and answer questions about data as they apply their growing understanding of numbers and the operations of addition and subtraction. They extend their spatial understanding of Core Content in geometry developed in kindergarten and grade one by solving problems involving two- and three-dimensional geometric figures. Students are introduced to a few critical concepts that will become Core Content in grade three. Specifically, they begin to work with multiplication and division and learn what a fraction is.

Performance Expectations	*Explanatory Comments and Examples*
Students are expected to:	
2.4.A Solve problems involving properties of two- and three-dimensional figures.	A critical component in the development of students' spatial and geometric understanding is the ability to solve problems involving the properties of figures. At the primary level, students must move from judging plane and space shapes by their appearance as whole shapes to focusing on the relationship of the sides, angles, or faces. At the same time, students must learn the language important for describing shapes according to their essential characteristics. Later, they will describe properties of shapes in more formal ways as they progress in geometry. Examples: • How many different ways can you fill the outline of the figure with pattern blocks? What is the greatest number of blocks you can use? The least number? Can you fill the outline with every whole number of blocks between the least number of blocks and the greatest number of blocks? • Build a figure or design out of five blocks. Describe it clearly enough so that someone else could build it without seeing it. Blocks may represent two-dimensional figures (i.e., pattern blocks) or three-dimensional figures (i.e., wooden geometric solids).
2.4.B Collect, organize, represent, and interpret data in bar graphs and picture graphs.	In a picture graph, a single picture represents a single object. Pictographs, where a symbol represents more than one unit, are introduced in grade three when multiplication skills are developed.

Students are expected to:

2.4.C	Model and describe multiplication situations in which sets of equal size are joined.	Multiplication is introduced in grade two only at a conceptual level. This is a foundation for the more systematic study of multiplication in grade three. Small numbers should be used in multiplication problems that are posed for students in grade two.

Example:

- You have 4 boxes with 3 apples in each box. How many apples do you have?

2.4.D	Model and describe division situations in which sets are separated into equal parts.	Division is introduced in grade two only at a conceptual level. This is a foundation for the more systematic study of division in grade three. Small numbers should be used in division problems that are posed for students in grade two.

Example:

- You have 15 apples to share equally among 5 classmates. How many apples will each classmate get?

2.4.E	Interpret a fraction as a number of equal parts of a whole or a set.	Examples:

- Juan, Chan, and Hortense are going to share a large cookie in the shape of a circle. Draw a picture that shows how you can cut up the cookie in three fair shares, and tell how big each piece is as a fraction of the whole cookie.

- Ray has two blue crayons, one red crayon, and one yellow crayon. What fraction of Ray's crayons is red? What fraction of the crayons is blue?

Grade 2

2.5. Core Processes: *Reasoning, problem solving, and communication*

Students further develop the concept that doing mathematics involves solving problems and talking about what they did to solve those problems. Second-grade problems emphasize addition and subtraction with increasingly large numbers, measurement, and early concepts of multiplication and division. Students communicate their mathematical thinking and make increasingly more convincing mathematical arguments. Students participate in mathematical discussions involving questions like "How did you get that?"; "Why did you use that strategy?"; and "Why is that true?" Students continue to build their mathematical vocabulary as they use correct mathematical language appropriate to grade two when discussing and refining solutions to problems.

Performance Expectations	Explanatory Comments and Examples
Students are expected to:	
2.5.A Identify the question(s) asked in a problem and any other questions that need to be answered in order to solve the problem.	Descriptions of solution processes and explanations can include numbers, words (including mathematical language), pictures, or physical objects. Students should be able to use all of these representations as needed. For a particular solution, students should be able to explain or show their work using at least one of these representations and verify that their answer is reasonable.
2.5.B Identify the given information that can be used to solve a problem.	
2.5.C Recognize when additional information is required to solve a problem.	
2.5.D Select from a variety of problem-solving strategies and use one or more strategies to solve a problem.	Examples:
	• A bag full of jellybeans is on the table. There are 10 black jellybeans in the bag. There are twice as many red jellybeans as black jellybeans. There are 2 fewer red jellybeans than yellow jellybeans. There are half as many pink jellybeans as yellow jellybeans. How many jellybeans are in the bag? Explain your answer.
2.5.E Identify the answer(s) to the question(s) in a problem.	
2.5.F Describe how a problem was solved.	
2.5.G Determine whether a solution to a problem is reasonable.	• Suzy, Ben, and Pedro have found 1 quarter, 1 dime, and 4 pennies under the sofa. Their mother has lots of change in her purse, so they could trade any of these coins for other coins adding up to the same value. She says they can keep the money if they can tell her what coins they need so the money can be shared equally among them. How can they do this?

Grade 3

3.1. Core Content: *Addition, subtraction, and place value*   *(Numbers, Operations)*

Students solidify and formalize important concepts and skills related to addition and subtraction. In particular, students extend critical concepts of the base ten number system to include large numbers, they formalize procedures for adding and subtracting large numbers, and they apply these procedures in new contexts.

Performance Expectations	Explanatory Comments and Examples
Students are expected to:	
3.1.A Read, write, compare, order, and represent numbers to 10,000 using numbers, words, and symbols.	This expectation reinforces and extends place value concepts. Symbols used to describe comparisons include <, >, =. Examples: • Fill in the box with <, >, or = to make a true sentence: 3,546 □ 4,356. • Is 5,683 closer to 5,600 or 5,700?
3.1.B Round whole numbers through 10,000 to the nearest ten, hundred, and thousand.	Example: • Round 3,465 to the nearest ten and then to the nearest hundred.
3.1.C Fluently and accurately add and subtract whole numbers using the standard regrouping algorithms.	Teachers should be aware that in some countries the algorithms might be recorded differently.
3.1.D Estimate sums and differences to approximate solutions to problems and determine reasonableness of answers.	Example: • Marla has $10 and plans to spend it on items priced at $3.72 and $6.54. Use estimation to decide whether Marla's plan is a reasonable one, and justify your answer.
3.1.E Solve single- and multi-step word problems involving addition and subtraction of whole numbers and verify the solutions.	The intent of this expectation is for students to show their work, explain their thinking, and verify that the answer to the problem is reasonable in terms of the original context and the mathematics used to solve the problem. Verifications can include the use of numbers, words, pictures, or equations.

Grade 3

3.2. Core Content: Concepts of multiplication and division   *(Operations, Algebra)*

Students learn the meaning of multiplication and division and how these operations relate to each other. They begin to learn multiplication and division facts and how to multiply larger numbers. Students use what they are learning about multiplication and division to solve a variety of problems. With a solid understanding of these two key operations, students are prepared to formalize the procedures for multiplication and division in grades four and five.

Performance Expectations	*Explanatory Comments and Examples*
Students are expected to:	
3.2.A Represent multiplication as repeated addition, arrays, counting by multiples, and equal jumps on the number line, and connect each representation to the related equation.	Students should be familiar with using words, pictures, physical objects, and equations to represent multiplication. They should be able to connect various representations of multiplication to the related multiplication equation. Representing multiplication with arrays is a precursor to more formalized area models for multiplication developed in later grades beginning with grade four.

The equation $3 \times 4 = 12$ could be represented in the following ways:
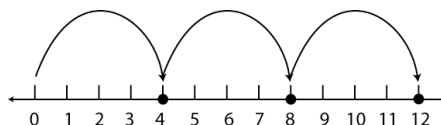
— Equal sets:

— An array:

— Repeated addition: $4 + 4 + 4$

— Three equal jumps forward from 0 on the number line to 12:

3.2.B Represent division as equal sharing, repeated subtraction, equal jumps on the number line, and formation of equal groups of objects, and connect each representation to the related equation.

Students should be familiar with using words, pictures, physical objects, and equations to represent division. They should be able to connect various representations of division to the related equation.

Division can model both equal sharing (how many in each group) and equal groups (how many groups).

The equation $12 \div 4 = 3$ could be represented in the following ways:
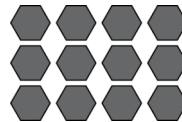
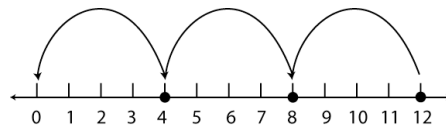— Equal groups: Equal sharing:

— An array:

— Repeated subtraction: The expression $12 - 4 - 4 - 4$ involves 3 subtractions of 4.

— Three equal jumps backward from 12 to 0 on the number line:

3.2.C Determine products, quotients, and missing factors using the inverse relationship between multiplication and division.
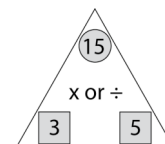
Example:

- To find the value of *N* in $3 \times N = 18$, think $18 \div 3 = 6$.

Students can use multiplication and division fact families to understand the inverse relationship between multiplication and division.

Examples:

- $3 \times 5 = 15$ $5 \times 3 = 15$
  $15 \div 3 = 5$ $15 \div 5 = 3$

3.2.D Apply and explain strategies to compute multiplication facts to 10 X 10 and the related division facts.

Strategies for multiplication include skip counting (repeated addition), fact families, double-doubles (when 4 is a factor), "think ten" (when 9 is a factor, think of it as $10 - 1$), and decomposition of arrays into smaller known parts.

Number properties can be used to help remember basic facts.

$5 \times 3 = 3 \times 5$ (Commutative Property)
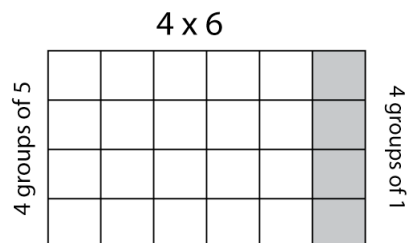
$1 \times 5 = 5 \times 1 = 5$ (Identity Property)

$0 \times 5 = 5 \times 0 = 0$ (Zero Property)

$5 \times 6 = 5 \times (2 \times 3) = (5 \times 2) \times 3 = 10 \times 3 = 30$ (Associative Property)

$4 \times 6 = 4 (5 + 1) = (4 \times 5) + (4 \times 1) = 20 + 4 = 24$ (Distributive Property)

4 x 6

Division strategies include using fact families and thinking of missing factors.

3.2.E Quickly recall those multiplication facts for which one factor is 1, 2, 5, or 10 and the related division facts.

Many students will learn all of the multiplication facts to 10 X 10 by the end of third grade, and all students should be given the opportunity to do so.

3.2.F Solve and create word problems that match multiplication or division equations.

The goal is for students to be able to represent multiplication and division sentences with an appropriate situation, using objects, pictures, or written or spoken words. This standard is about helping students connect symbolic representations to the situations they model. While some students may create word problems that are detailed or lengthy, this is not necessary to meet the expectation. Just as we want students to be able to translate 5 groups of 3 cats into 5 x 3 = 15; we want students to look at an equation like $12 \div 4 = 3$ and connect it to a situation using objects, pictures, or words.

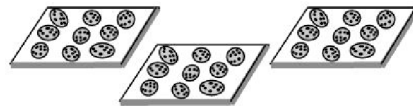*Students are expected to:*

Example:

- Equation: $3 \times 9 = ?$

  [Problem situation:

  There are 3 trays of cookies with 9 cookies on each tray. How many cookies are there in all?]

3.2.G Multiply any number from 11 through 19 by a single-digit number using the distributive property and place value concepts.

Example:

- 6×12 can be thought of as 6 tens and 6 twos, which equal 60 and 12, totaling 72.

	10	2
6	6 x 10 = 60	6 x 2 = 12

6 groups of 10   6 groups of 2

3.2.H Solve single- and multi-step word problems involving multiplication and division and verify the solutions.
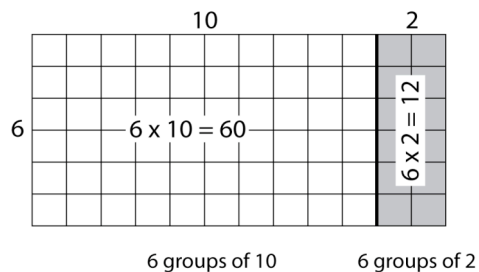
Problems include using multiplication to determine the number of possible combinations or outcomes for a situation, and division contexts that require interpretations of the remainder.

The intent of this expectation is for students to show their work, explain their thinking, and verify that the answer to the problem is reasonable in terms of the original context and the mathematics used to solve the problem. Verifications can include the use of numbers, words, pictures, physical objects, or equations.

Examples:

- Determine the number of different outfits that can be made with four shirts and three pairs of pants.

- There are 14 soccer players on the boys' team and 13 on the girls' team. How many vans are needed to take all players to the soccer tournament if each van can take 5 players?

Grade 3

3.3. Core Content: *Fraction concepts* *(Numbers, Algebra)*

Students learn about fractions and how they are used. Students deepen their understanding of fractions by comparing and ordering fractions and by representing them in different ways. With a solid knowledge of fractions as numbers, students are prepared to be successful when they add, subtract, multiply, and divide fractions to solve problems in later grades.

Performance Expectations	Explanatory Comments and Examples
Students are expected to:	
3.3.A Represent fractions that have denominators of 2, 3, 4, 5, 6, 8, 9, 10, and 12 as parts of a whole, parts of a set, and points on the number line.	The focus is on numbers less than or equal to 1. Students should be familiar with using words, pictures, physical objects, and equations to represent fractions.
3.3.B Compare and order fractions that have denominators of 2, 3, 4, 5, 6, 8, 9, 10, and 12.	Fractions can be compared using benchmarks (such as $\frac{1}{2}$ or 1), common numerators, or common denominators. Symbols used to describe comparisons include <, >, =.
	Fractions with common denominators may be compared and ordered using the numerators as a guide.
	$$\frac{2}{6} < \frac{3}{6} < \frac{5}{6}$$
	Fractions with common numerators may be compared and ordered using the denominators as a guide.
	$$\frac{3}{10} < \frac{3}{8} < \frac{3}{4}$$
	Fractions may be compared using $\frac{1}{2}$ as a benchmark.
3.3.C Represent and identify equivalent fractions with denominators of 2, 3, 4, 5, 6, 8, 9, 10, and 12.	Students could represent fractions using the number line, physical objects, pictures, or numbers.
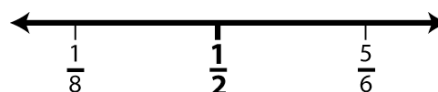
3.3.D Solve single- and multi-step word problems involving comparison of fractions and verify the solutions.

The intent of this expectation is for students to show their work, explain their thinking, and verify that the answer to the problem is reasonable in terms of the original context and the mathematics used to solve the problem. Verifications can include the use of numbers, words, pictures, physical objects, or equations.

Examples:

- Emile and Jordan ordered a medium pizza. Emile ate $\frac{1}{3}$ of it and Jordan ate $\frac{1}{4}$ of it. Who ate more pizza? Explain how you know.

- Janie and Li bought a dozen balloons. Half of them were blue, $\frac{1}{3}$ were white, and $\frac{1}{6}$ were red. Were there more blue, red, or white balloons? Justify your answer.

Grade 3
3.4. Core Content: Geometry   *(Geometry/Measurement)*

Students learn about lines and use lines, line segments, and right angles as they work with quadrilaterals. Students connect this geometric work to numbers, operations, and measurement as they determine simple perimeters in ways they will use when calculating perimeters of more complex figures in later grades.

Performance Expectations	*Explanatory Comments and Examples*
Students are expected to:	
3.4.A Identify and sketch parallel, intersecting, and perpendicular lines and line segments.	
3.4.B Identify and sketch right angles.	
3.4.C Identify and describe special types of quadrilaterals.	Special types of quadrilaterals include squares, rectangles, parallelograms, rhombi, trapezoids and kites.
3.4.D Measure and calculate perimeters of quadrilaterals.	Example: • Sketch a parallelogram with two sides 9 cm long and two sides 6 cm long. What is the perimeter of the parallelogram?
3.4.E Solve single- and multi-step word problems involving perimeters of quadrilaterals and verify the solutions.	Examples: • Julie and Jacob have recently created two rectangular vegetable gardens in their backyard. One garden measures 6 ft by 8 ft, and the other garden measures 10 ft by 5 ft. They decide to place a small fence around the outside of each garden to prevent their dog from getting into their new vegetables. How many feet of fencing should Julie and Jacob buy to fence both gardens?

Grade 3

3.5. Additional Key Content   *(Algebra, Geometry/Measurement, Data/Statistics/Probability)*

Students solidify and formalize a number of important concepts and skills related to Core Content studied in previous grades. In particular, students demonstrate their understanding of equivalence as an important foundation for later work in algebra. Students also reinforce their knowledge of measurement as they use standard units for temperature, weight, and capacity. They continue to develop data organization skills as they reinforce multiplication and division concepts with a variety of types of graphs.

Performance Expectations	Explanatory Comments and Examples
Students are expected to:	
3.5.A Determine whether two expressions are equal and use "=" to denote equality.	Examples: • Is $5 \times 3 = 3 \times 5$ a true statement? • Is $24 \div 3 = 2 \times 4$ a true statement? A common error students make is using the mathematical equivalent of a run-on sentence to solve some problems—students carry an equivalence from a previous expression into a new expression with an additional operation. For example, when adding $3 + 6 + 7$, students sometimes incorrectly write: $3 + 6 = 9 + 7 = 16$ Correct sentences: $3 + 6 = 9$ $9 + 7 = 16$
3.5.B Measure temperature in degrees Fahrenheit and degrees Celsius using a thermometer.	The scale on a thermometer is essentially a vertical number line. Students may informally deal with negative numbers in this context, although negative numbers are not formally introduced until grade six. Measure temperature to the nearest degree.
3.5.C Estimate, measure, and compare weight and mass using appropriate-sized U.S. customary and metric units.	
3.5.D Estimate, measure, and compare capacity using appropriate-sized U.S. customary and metric units.	

Performance Expectations

Students are expected to:

3.5.E   Construct and analyze pictographs, frequency tables, line plots, and bar graphs.

Explanatory Comments and Examples

Students can write questions to be answered with information from a graph. Graphs and tables can be used to compare sets of data.

Using pictographs in which a symbol stands for multiple objects can reinforce the development of both multiplication and division skills. Determining appropriate scale and units for the axes of various types of graphs can also reinforce multiplication and division skills.

Grade 3

3.6. Core Processes: *Reasoning, problem solving, and communication*

Students in grade three solve problems that extend their understanding of core mathematical concepts—such as geometric figures, fraction concepts, and multiplication and division of whole numbers—as they make strategic decisions that bring them to reasonable solutions. Students use pictures, symbols, or mathematical language to explain the reasoning behind their decisions and solutions. They further develop their problem-solving skills by making generalizations about the processes used and applying these generalizations to similar problem situations. These critical reasoning, problem-solving, and communication skills represent the kind of mathematical thinking that equips students to use the mathematics they know to solve a growing range of useful and important problems and to make decisions based on quantitative information.

Performance Expectations	*Explanatory Comments and Examples*
Students are expected to:	
3.6.A Determine the question(s) to be answered given a problem situation.	Descriptions of solution processes and explanations can include numbers, words (including mathematical language), pictures, physical objects, or equations. Students should be able to use all of these representations as needed. For a particular solution, students should be able to explain or show their work using at least one of these representations and verify that their answer is reasonable.
3.6.B Identify information that is given in a problem and decide whether it is necessary or unnecessary to the solution of the problem.	
3.6.C Identify missing information that is needed to solve a problem.	
3.6.D Determine whether a problem to be solved is similar to previously solved problems, and identify possible strategies for solving the problem.	Examples:
3.6.E Select and use one or more appropriate strategies to solve a problem.	• Whitney wants to put a fence around the perimeter of her square garden. She plans to include a gate that is 3 ft wide. The length of one side of the garden is 19 ft. The fencing comes in two sizes: rolls that are 18 ft long and 24 ft long. Which rolls and how many of each should Whitney buy in order to have the least amount of leftover fencing? Justify your answer.
3.6.F Represent a problem situation using words, numbers, pictures, physical objects, or symbols.	
3.6.G Explain why a specific problem-solving strategy or procedure was used to determine a solution.	• A soccer team is selling water bottles with soccer balls painted on them to raise money for new equipment. The team bought 10 boxes of water bottles. Each box cost $27 and had 9 bottles. At what price should the team sell each bottle in order to make $180 profit to pay for new soccer balls? Justify your answer.
3.6.H Analyze and evaluate whether a solution is reasonable, is mathematically correct, and answers the question.	
3.6.I Summarize mathematical information, draw conclusions, and explain reasoning.	
3.6.J Make and test conjectures based on data (or information) collected from explorations and experiments.	

Grade 4

4.1. Core Content: *Multi-digit multiplication* (Numbers, Operations, Algebra)

Students learn basic multiplication facts and efficient procedures for multiplying two- and three-digit numbers. They explore the relationship between multiplication and division as they learn related division and multiplication facts in the same fact family. These skills, along with mental math and estimation, allow students to solve problems that call for multiplication. Building on an understanding of how multiplication and division relate to each other, students prepare to learn efficient procedures for division, which will be developed in fifth grade. Multiplication of whole numbers is not only a basic skill, it is also closely connected to Core Content in this grade level on area, and this connection reinforces understanding of both concepts. Multiplication is also central to students' study of many other topics in mathematics across the grades, including fractions, volume, and algebra.

Performance Expectations	*Explanatory Comments and Examples*
Students are expected to:	
4.1.A Quickly recall multiplication facts through 10 X 10 and the related division facts.	
4.1.B Identify factors and multiples of a number.	Examples: • The factors of 12 are 1, 2, 3, 4, 6, 12. • The multiples of 12 are 12, 24, 36, 48, . . .
4.1.C Represent multiplication of a two-digit number by a two-digit number with place value models.	Representations can include pictures or physical objects, or students can describe the process in words (14 times 16 is the same as 14 times 10 added to 14 times 6). The algorithm for multiplication is addressed in expectation 4.1.F. Example: • $14 \times 16 = 224$ $100 + 40 + 60 + 24 = 224$
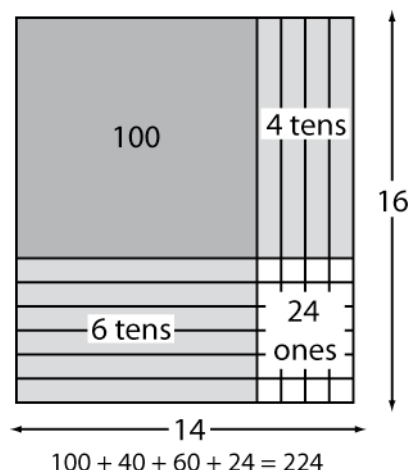
Performance Expectations	*Explanatory Comments and Examples*
Students are expected to:	

4.1.D Multiply by 10, 100, and 1,000.

Multiplying by 10, 100, and 1,000 extends place value concepts to large numbers through the millions. Students can use place value and properties of operations to determine these products.

Examples:

- $10 \times 5,000 = 50,000$

   $100 \times 5,000 = 500,000$

   $1,000 \times 5,000 = 5,000,000$

- 40×300

   $= (4 \times 10) \times (3 \times 100)$

   $= (4 \times 3) \times (10 \times 100)$

   $= 12 \times 1,000$

   $= 12,000$

4.1.E Compare the values represented by digits in whole numbers using place value.

Example:

- Compare the values represented by the digit 4 in 4,000,000 and 40,000. [The value represented by the 4 in the millions place is 100 times as much as the value represented by the 4 in the ten-thousands place.]

4.1.F Fluently and accurately multiply up to a three-digit number by one- and two-digit numbers using the standard multiplication algorithm.

Example:

- $$\begin{array}{r} 245 \\ \times\ 7 \end{array} \qquad \begin{array}{r} ^{3\ 3} \\ 245 \\ \times\ 7 \\ \hline 1715 \end{array}$$

Teachers should be aware that in some countries the algorithm might be recorded differently.

4.1.G Mentally multiply two-digit numbers by numbers through 10 and by multiples of 10.

Examples:

- $4 \times 32 = (4 \times 30) + (4 \times 2)$

- $4 \times 99 = 400 - 4$

- $25 \times 30 = 75 \times 10$

Performance Expectations	*Explanatory Comments and Examples*
Students are expected to:	

4.1.H   Estimate products to approximate solutions to problems and determine reasonableness of answers.

Example:

- 28×120 is approximately 30 times 100, so the product should be around 3,000.

4.1.I   Solve single- and multi-step word problems involving multi-digit multiplication and verify the solutions.

The intent of this expectation is for students to show their work, explain their thinking, and verify that the answer to the problem is reasonable in terms of the original context and the mathematics used to solve the problem. Verifications can include the use of numbers, words, pictures, or equations.

Problems could include multi-step problems that use operations other than multiplication.

4.1.J   Solve single- and multi-step word problems involving division and verify the solutions.

The intent of this expectation is for students to show their work, explain their thinking, and verify that the answer to the problem is reasonable in terms of the original context and the mathematics used to solve the problem. Verifications can include the use of numbers, words, pictures, or equations.

Division problems should reinforce connections between multiplication and division. The example below can be solved using multiplication along with some addition and subtraction.

Example:

- A class of 20 students shares a box containing 385 animal crackers. What is each student's equal share? How many crackers are left over?

Division algorithms, including long division, are developed in fifth grade.

Grade 4

4.2. Core Content: *Fractions, decimals, and mixed numbers*   *(Numbers, Algebra)*

Students solidify and extend their understanding of fractions (including mixed numbers) to include decimals and the relationships between fractions and decimals. Students work with common factors and common multiples as preparation for learning procedures for fraction operations in grades five and six. When they are comfortable with and knowledgeable about fractions, students are likely to be successful with the challenging skills of learning how to add, subtract, multiply, and divide fractions.

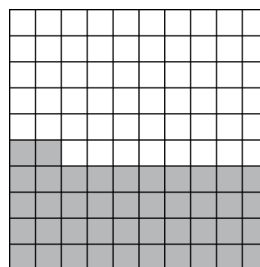| *Performance Expectations*	*Explanatory Comments and Examples*
Students are expected to:	
4.2.A Represent decimals through hundredths with place value models, fraction equivalents, and the number line.	Students should know how to write decimals and show them on the number line and should understand their mathematical connections to place value models and fraction equivalents. Students should be able to represent decimals with words, pictures, or physical objects, and connect these representations to the corresponding decimal.
4.2.B Read, write, compare, and order decimals through hundredths.	Decimals may be compared using benchmarks, such as 0, 0.5, 1, or 1.5. Decimals may also be compared using place value. Examples: • List in increasing order: 0.7, 0.2, 1.4. • Write an inequality that compares 0.05 and 0.50.
4.2.C Convert a mixed number to a fraction and vice versa, and visually represent the number.	Students should be able to use either the fraction or mixed-number form of a number as appropriate to a given situation, and they should be familiar with representing these numbers with words, pictures, and physical objects.
4.2.D Convert a decimal to a fraction and vice versa, and visually represent the number.	Students should be familiar with using pictures and physical objects to visually represent decimals and fractions. For this skill at this grade, fractions should be limited to those that are equivalent to fractions with denominators of 10 or 100. Examples: • $\frac{3}{10} = 0.3$

Students are expected to:

- $0.42 = \dfrac{42}{100}$

- $\dfrac{5}{20} = 0.25$

4.2.E	Compare and order decimals and fractions (including mixed numbers) on the number line, lists, and the symbols <, >, or =.

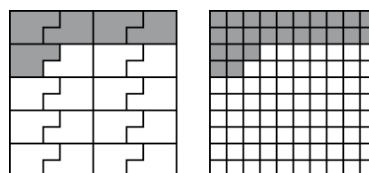Examples:

- Compare each pair of numbers using <, >, or =:

  $\dfrac{6}{10}\ \square\ 0.8$

  $1\dfrac{1}{2}\ \square\ \dfrac{3}{2}$

  $0.75\ \square\ \dfrac{1}{2}$

- Correctly show $\dfrac{3}{5}$, 0.35, $3\dfrac{1}{2}$ on the number line.

- Order the following numbers from least to greatest: $\dfrac{7}{6}$, 6.2, $\dfrac{1}{12}$, 0.88.

4.2.F	Write a fraction equivalent to a given fraction.

Example:

- Write at least two fractions equivalent to each fraction given below:

  $\dfrac{1}{2}, \dfrac{5}{6}, \dfrac{2}{3}$

4.2.G   Simplify fractions using common factors.

4.2.H   Round fractions and decimals to the nearest whole number.

4.2.I   Solve single- and multi-step word problems involving comparison of decimals and fractions (including mixed numbers), and verify the solutions.

The intent of this expectation is for students to show their work, explain their thinking, and verify that the answer to the problem is reasonable in terms of the original context and the mathematics used to solve the problem. Verifications can include the use of numbers, words, pictures, or equations.

Example:

- Ms. Ortiz needs $1\frac{1}{2}$ pounds of sliced turkey.

  She picked up a package labeled "1.12 lbs." Would she have enough turkey with this package? Explain why or why not.

Grade 4

4.3. Core Content: *Concept of area*   *(Geometry/Measurement, Algebra)*

Students learn how to find the area of a rectangle as a basis for later work with areas of other geometric figures. They select appropriate units, tools, and strategies, including formulas, and use them to solve problems involving perimeter and area. Solving such problems helps students develop spatial skills, which are critical for dealing with a wide range of geometric concepts. The study of area is closely connected to Core Content on multiplication, and connections between these concepts should be emphasized whenever possible.

Performance Expectations	*Explanatory Comments and Examples*
Students are expected to:	
4.3.A Determine congruence of two-dimensional figures.	At this grade level, students determine congruence primarily by making direct comparisons (i.e., tracing or cutting). They may also use informal notions of transformations described as flips, turns, and slides. Both the language and the concepts of transformations are more formally developed in grade eight.
4.3.B Determine the approximate area of a figure using square units.	Example: • Draw a rectangle 3.5 cm by 6 cm on centimeter grid paper. About how many squares fit inside the rectangle? • Cover a footprint with square tiles or outline it on grid paper. About how many squares fit inside the footprint?
4.3.C Determine the perimeter and area of a rectangle using formulas, and explain why the formulas work.	This is an opportunity to connect area to the concept of multiplication, a useful model for multiplication that extends into algebra. Students should also work with squares as special rectangles. Example: • Outline on grid paper a rectangle that is 4 units long and 3 units wide. Without counting the squares, how can you determine the area? Other than measuring, how could you use a shortcut to find the perimeter of the rectangle?
4.3.D Determine the areas of figures that can be broken down into rectangles.	Example: • Find the area of each figure:
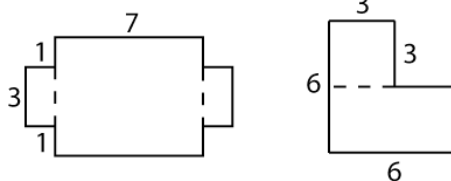
Performance Expectations

Students are expected to:

4.3.E Demonstrate that rectangles with the same area can have different perimeters, and that rectangles with the same perimeter can have different areas.

4.3.F Solve single- and multi-step word problems involving perimeters and areas of rectangles and verify the solutions.

Explanatory Comments and Examples

Example:

- Draw different rectangles, each with an area of 24 square units, and compare their perimeters. What patterns do you notice in the data? Record your observations.

The intent of this expectation is for students to show their work, explain their thinking, and verify that the answer to the problem is reasonable in terms of the original context and the mathematics used to solve the problem. Verifications can include the use of numbers, words, pictures, or equations.

Problems include those involving U.S. customary and metric units, including square units.

Grade 4

4.4. Additional Key Content   *(Geometry/Measurement, Algebra, Data/Statistics/Probability)*

Students use coordinate grids to connect numbers to basic ideas in algebra and geometry. This connection between algebra and geometry runs throughout advanced mathematics and allows students to use tools from one branch of mathematics to solve problems related to another branch. Students also extend and reinforce their work with whole numbers and fractions to describe sets of data and find simple probabilities. Students combine measurement work with their developing ideas about multiplication and division as they do basic measurement conversions. They begin to use algebraic notation while solving problems in preparation for formalizing algebraic thinking in later grades.

Performance Expectations	**Explanatory Comments and Examples**
Students are expected to:	
4.4.A Represent an unknown quantity in simple expressions, equations, and inequalities using letters, boxes, and other symbols.	Example: • There are 5 jars. Lupe put the same number of marbles in each jar. Write an equation or expression that shows how many marbles are in each jar if there are 40 marbles total. [$5 \times \square = 40$ or $5 \times M = 40$; M represents the number of marbles]
4.4.B Solve single- and multi-step problems involving familiar unit conversions, including time, within either the U.S. customary or metric system.	Examples: • Jill bought 3 meters of ribbon and cut it into pieces 25 centimeters long. How many 25-centimeter pieces of ribbon did she have? • How many quarts of lemonade are needed to make 25 one-cup servings?
4.4.C Estimate and determine elapsed time using a calendar, a digital clock, and an analog clock.	
4.4.D Graph and identify points in the first quadrant of the coordinate plane using ordered pairs.	Example:

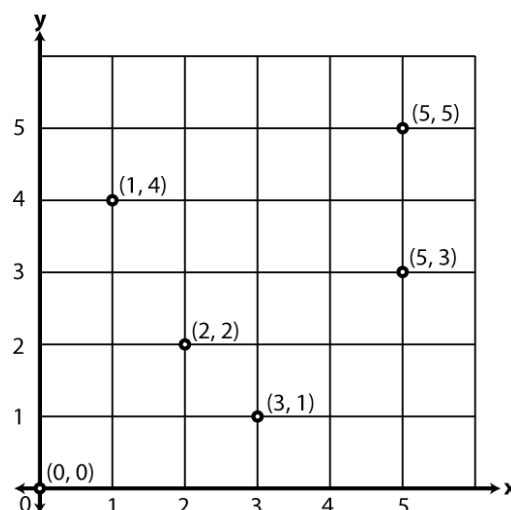| *Performance Expectations*	*Explanatory Comments and Examples*
Students are expected to:	

4.4.E Determine the median, mode, and range of a set of data and describe what each measure indicates about the data.

Example:

- What is the median number of siblings that students in this class have? What is the mode of the data? What is the range of the number of siblings? What does each of these values tell you about the students in the class?

Siblings of Class 4A

Number of Siblings of Class 4A

4.4.F Describe and compare the likelihood of events.
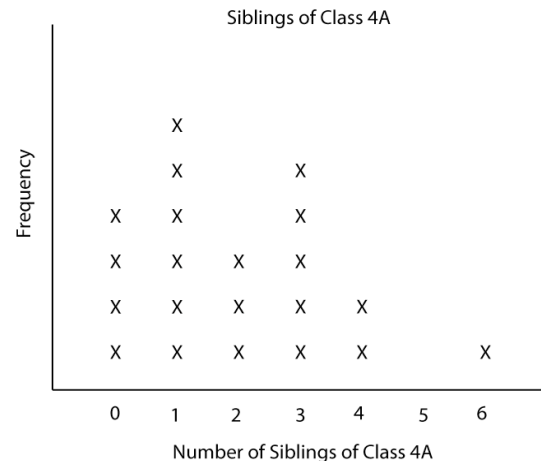
For this introduction to probability, an event can be described as *certain*, *impossible*, *likely*, or *unlikely*. Two events can be compared as *equally likely*, *not equally likely*, or as one being *more likely* or *less likely* than the other.

4.4.G Determine a simple probability from a context that includes a picture.

Probability is expressed as a number from 0 to 1.

Example:

- What is the probability of a blindfolded person choosing a black marble from the bowl?

4.4.H Display the results of probability experiments and interpret the results.
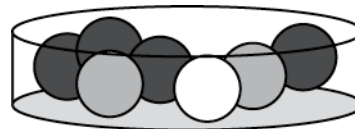
Displays include tallies, frequency tables, graphs, pictures, and fractions.

Grade 4

4.5. Core Processes: *Reasoning, problem solving, and communication*

Students in grade four solve problems that extend their understanding of core mathematical concepts—such as multiplication of multi-digit numbers, area, probability, and the relationships between fractions and decimals—as they make strategic decisions that bring them to reasonable solutions. Students use pictures, symbols, or mathematical language to explain the reasoning behind their decisions and solutions. They further develop their problem-solving skills by making generalizations about the processes used and applying these generalizations to similar problem situations. These critical reasoning, problem-solving, and communication skills represent the kind of mathematical thinking that equips students to use the mathematics they know to solve a growing range of useful and important problems and to make decisions based on quantitative information.

Performance Expectations	Explanatory Comments and Examples
Students are expected to:	
4.5.A Determine the question(s) to be answered given a problem situation.	Descriptions of solution processes and explanations can include numbers, words (including mathematical language), pictures, physical objects, or equations. Students should be able to use all of these representations as needed. For a particular solution, students should be able to explain or show their work using at least one of these representations and verify that their answer is reasonable.
4.5.B Identify information that is given in a problem and decide whether it is essential or extraneous to the solution of the problem.	
4.5.C Identify missing information that is needed to solve a problem.	
4.5.D Determine whether a problem to be solved is similar to previously solved problems, and identify possible strategies for solving the problem.	Examples:
4.5.E Select and use one or more appropriate strategies to solve a problem and explain why that strategy was chosen.	• Jake's family adopted a small dog, Toto. They have a rectangular dog pen that is 10 feet by 20 feet. Toto needs only half that area, so Jake plans to make the pen smaller by cutting each dimension in half. Jake's mother asked him to rethink his plan or Toto won't have the right amount of space.
4.5.F Represent a problem situation using words, numbers, pictures, physical objects, or symbols.	— Whose reasoning is correct—Jake's or his mother's? Why?
4.5.G Explain why a specific problem-solving strategy or procedure was used to determine a solution.	— According to Jake's plan, what fractional part of the old pen will be the area of the new pen? Give the answer in simplest form.
4.5.H Analyze and evaluate whether a solution is reasonable, is mathematically correct, and answers the question.	— Make a new plan so that the area of the new pen is half the area of the old pen.
4.5.I Summarize mathematical information, draw conclusions, and explain reasoning.	• The city is paying for a new deck around the community pool. The rectangular pool measures 50 meters by 25 meters. The deck, which will measure 5 meters wide, will surround the pool like a picture frame. If the cost of the deck is $25 for each square meter, what will be the total cost for the new deck? Explain your solution.
4.5.J Make and test conjectures based on data (or information) collected from explorations and experiments.	

Grade 5

5.1. Core Content: *Multi-digit division* *(Operations, Algebra)*

Students learn efficient ways to divide whole numbers. They apply what they know about division to solve problems, using estimation and mental math skills to decide whether their results are reasonable. This emphasis on division gives students a complete set of tools for adding, subtracting, multiplying, and dividing whole numbers—basic skills for everyday life and further study of mathematics.

Performance Expectations	*Explanatory Comments and Examples*
Students are expected to:	
5.1.A Represent multi-digit division using place value models and connect the representation to the related equation.	Students use pictures or grid paper to represent division and describe how that representation connects to the related equation. They could also use physical objects such as base ten blocks to support the visual representation. Note that the algorithm for long division is addressed in expectation 5.1.C.
5.1.B Determine quotients for multiples of 10 and 100 by applying knowledge of place value and properties of operations.	Example: • Using the fact that $16 \div 4 = 4$, students can generate the related quotients $160 \div 4 = 40$ and $160 \div 40 = 4$.
5.1.C Fluently and accurately divide up to a four-digit number by one- or two-digit divisors using the standard long-division algorithm.	The use of 'R' or 'r' to indicate a remainder may be appropriate in most of the examples students encounter in grade five. However, students should also be aware that in subsequent grades, they will learn additional ways to represent remainders, such as fractional or decimal parts. Example: • $$\begin{array}{r} 132 \text{ r } 1 \\ 6\overline{)793} \\ -6 \\ \hline 19 \\ -18 \\ \hline 13 \\ -12 \\ \hline 1 \end{array}$$ Teachers should be aware that in some countries the algorithm might be recorded differently.

Performance Expectations

Students are expected to:

Explanatory Comments and Examples

5.1.D Estimate quotients to approximate solutions and determine reasonableness of answers in problems involving up to two-digit divisors.

Example:

- The team has saved $45 to buy soccer balls. If the balls cost $15.95 each, is it reasonable to think there is enough money for more than two balls?

Problems like 54,596 ÷ 798, which can be estimated by 56,000 ÷ 800, while technically beyond the standards, could be included when appropriate. The numbers are easily manipulated and the problems support the ongoing development of place value.

5.1.E Mentally divide two-digit numbers by one-digit divisors and explain the strategies used.

5.1.F Solve single- and multi-step word problems involving multi-digit division and verify the solutions.

The intent of this expectation is for students to show their work, explain their thinking, and verify that the answer to the problem is reasonable in terms of the original context and the mathematics used to solve the problem. Verifications can include the use of numbers, words, pictures, or equations.

Problems include those with and without remainders.

Grade 5

5.2. Core Content: *Addition and subtraction of fractions and decimals*   (Numbers, Operations, Algebra)

Students extend their knowledge about adding and subtracting whole numbers to learning procedures for adding and subtracting fractions and decimals. Students apply these procedures, along with mental math and estimation, to solve a wide range of problems that involve more of the types of numbers students see in other school subjects and in their lives.

Performance Expectations	Explanatory Comments and Examples

Students are expected to:

5.2.A   Represent addition and subtraction of fractions and mixed numbers using visual and numerical models, and connect the representation to the related equation.

This expectation includes numbers with like and unlike denominators. Students should be able to show these operations on a number line and should be familiar with the use of pictures and physical materials (like fraction pieces or fraction bars) to represent addition and subtraction of mixed numbers. They should be able to describe how a visual representation connects to the related equation.

Example:
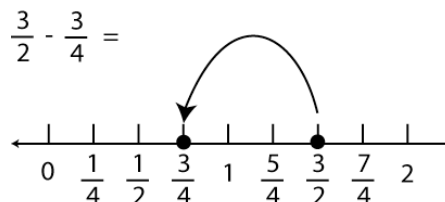
5.2.B   Represent addition and subtraction of decimals using place value models and connect the representation to the related equation.

Students should be familiar with using pictures and physical objects to represent addition and subtraction of decimals and be able to describe how those representations connect to related equations. Representations may include base ten blocks, number lines, and grid paper.

5.2.C   Given two fractions with unlike denominators, rewrite the fractions with a common denominator.

Fraction pairs include denominators with and without common factors.

When students are fluent in writing equivalent fractions, it helps them compare fractions and helps prepare them to add and subtract fractions.

Examples:

- Write equivalent fractions with a common denominator for $\frac{2}{3}$ and $\frac{3}{4}$.

- Write equivalent fractions with a common denominator for $\frac{3}{8}$ and $\frac{1}{6}$.

5.2.D	Determine the greatest common factor and the least common multiple of two or more whole numbers.	Least common multiple (LCM) can be used to determine common denominators when adding and subtracting fractions. Greatest common factor (GCF) can be used to simplify fractions.
5.2.E	Fluently and accurately add and subtract fractions, including mixed numbers.	Fractions can be in either proper or improper form. Students should also be able to work with whole numbers as part of this expectation.
5.2.F	Fluently and accurately add and subtract decimals.	Students should work with decimals less than 1 and greater than 1, as well as whole numbers, as part of this expectation.
5.2.G	Estimate sums and differences of fractions, mixed numbers, and decimals to approximate solutions to problems and determine reasonableness of answers.	Example: • Jared is making a frame for a picture that is $10\frac{3}{4}$ inches wide and $15\frac{1}{8}$ inches tall. He has a 4-ft length of metal framing material. Estimate whether he will have enough framing material to frame the picture.
5.2.H	Solve single- and multi-step word problems involving addition and subtraction of whole numbers, fractions (including mixed numbers), and decimals, and verify the solutions.	The intent of this expectation is for students to show their work, explain their thinking, and verify that the answer to the problem is reasonable in terms of the original context and the mathematics used to solve the problem. Verifications can include the use of numbers, words, pictures, or equations. Multi-step problems may also include previously learned computational skills like multiplication and division of whole numbers.

Grade 5

5.3. Core Content: *Triangles and quadrilaterals*   *(Geometry/Measurement, Algebra)*

Students focus on triangles and quadrilaterals to formalize and extend their understanding of these geometric shapes. They classify different types of triangles and quadrilaterals and develop formulas for their areas. In working with these formulas, students reinforce an important connection between algebra and geometry. They explore symmetry of these figures and use what they learn about triangles and quadrilaterals to solve a variety of problems in geometric contexts.

Performance Expectations	*Explanatory Comments and Examples*
Students are expected to:	
5.3.A Classify quadrilaterals.	Students sort a set of quadrilaterals into their various types, including parallelograms, kites, squares, rhombi, trapezoids, and rectangles, noting that a square can also be classified as a rectangle, parallelogram, and rhombus.
5.3.B Identify, sketch, and measure acute, right, and obtuse angles.	Example: • Use a protractor to measure the following angles and label each as acute, right, or obtuse.
5.3.C Identify, describe, and classify triangles by angle measure and number of congruent sides.	Students classify triangles by their angle size using the terms *acute*, *right*, or *obtuse*. Students classify triangles by the length of their sides using the terms *scalene*, *isosceles*, or *equilateral*.
5.3.D Determine the formula for the area of a parallelogram by relating it to the area of a rectangle.	Students relate the area of a parallelogram to the area of a rectangle, as shown below.
5.3.E Determine the formula for the area of a triangle by relating it to the area of a parallelogram.	Students relate the area of a triangle to the area of a parallelogram, as shown below.
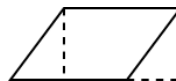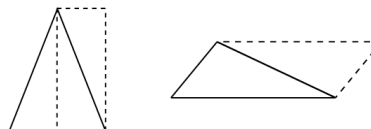
5.3.F	Determine the perimeters and areas of triangles and parallelograms.	Students may be given figures showing some side measures or may be expected to measure sides of figures. If students are not given side measures, but instead are asked to make their own measurements, it is important to discuss the approximate nature of any measurement.

5.3.G Draw quadrilaterals and triangles from given information about sides and angles.

Examples:

- Draw a triangle with one right angle and no congruent sides.

- Draw a rhombus that is not a square.

- Draw a right scalene triangle.

5.3.H Determine the number and location of lines of symmetry in triangles and quadrilaterals.

Example:

- Draw and count all the lines of symmetry in the square and isosceles triangle below. (Lines of symmetry are shown as dotted lines.)
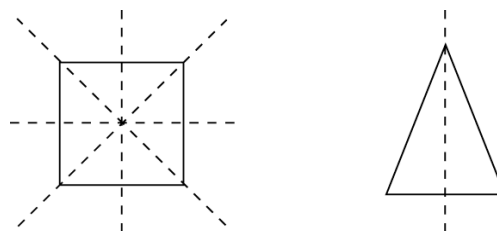
5.3.I Solve single- and multi-step word problems about the perimeters and areas of quadrilaterals and triangles and verify the solutions.

The intent of this expectation is for students to show their work, explain their thinking, and verify that the answer to the problem is reasonable in terms of the original context and the mathematics used to solve the problem. Verifications can include the use of numbers, words, pictures, or equations.

Grade 5

5.4. Core Content: *Representations of algebraic relationships*   *(Operations, Geometry/Measurement, Algebra)*

Students continue their development of algebraic thinking as they move toward more in-depth study of algebra in middle school. They use variables to write simple algebraic expressions describing patterns or solutions to problems. They use what they have learned about numbers and operations to evaluate simple algebraic expressions and to solve simple equations. Students make tables and graphs from linear equations to strengthen their understanding of algebraic relationships and to see the mathematical connections between algebra and geometry. These foundational algebraic skills allow students to see where mathematics, including algebra, can be used in real situations, and these skills prepare students for success in future grades.

Performance Expectations	*Explanatory Comments and Examples*
Students are expected to:	
5.4.A Describe and create a rule for numerical and geometric patterns and extend the patterns.	Example: • The picture shows a sequence of towers constructed from cubes. The number of cubes needed to build each tower forms a numeric pattern. Determine a rule for the number of cubes in each tower and use the rule to extend this pattern. Tower 1 Tower 2 Tower 3
5.4.B Write a rule to describe the relationship between two sets of data that are linearly related.	Rules can be written using words or algebraic expressions. Example: • The table below shows numerators (top row) and denominators (bottom row) of fractions equivalent to a given fraction ($\frac{1}{3}$). Write a rule that could be used to describe how the two rows could be related. <table><tr><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>3</td><td>6</td><td>9</td><td>?</td></tr></table>
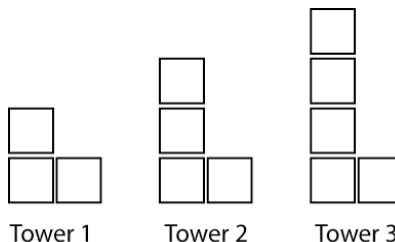
Performance Expectations	Explanatory Comments and Examples

Students are expected to:

5.4.C Write algebraic expressions that represent simple situations and evaluate the expressions, using substitution when variables are involved.

Students should evaluate expressions with and without parentheses. Evaluating expressions with parentheses is an initial step in learning the proper order of operations.

Examples:

- Evaluate $(4 \times n) + 5$ when $n = 2$.

- If 4 people can sit at 1 table, 8 people can sit at 2 tables, and 12 people can sit at 3 tables, and this relationship continues, write an expression to describe the number of people who can sit at n tables and tell how many people can sit at 67 tables.

- Compare the answers to A and B below. Why aren't the answers the same?

  A: $(3 \times 10) + 2$

  B: $3 \times (10 + 2)$

5.4.D Graph ordered pairs in the coordinate plane for two sets of data related by a linear rule and draw the line they determine.

Example:

- The table shows the total cost of purchasing different quantities of equally priced DVDs.

number purchased	0	2	5
total cost	$0	$10	$25

Graph the ordered pairs (0,0), (2,10), and (5, 25) and the line connecting the ordered pairs. Use the line to determine the total cost when 3 DVDs are purchased.

Cost and Number of DVDs Purchased
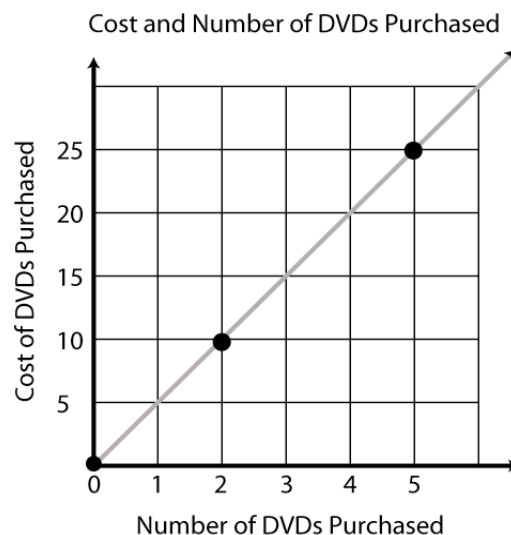
Grade 5

5.5. Additional Key Content *(Numbers, Data/Statistics/Probability)*

Students extend their work with common factors and common multiples as they deal with prime numbers. Students extend and reinforce their use of numbers, operations, and graphing to describe and compare data sets for increasingly complex situations they may encounter in other school subjects and in their lives.

Performance Expectations	*Explanatory Comments and Examples*
Students are expected to:	

5.5.A Classify numbers as prime or composite.

Divisibility rules can help determine whether a number has particular factors.

5.5.B Determine and interpret the mean of a small data set of whole numbers.

At this grade level, numbers for problems are selected so that the mean will be a whole number.

Example:

- Seven families report the following number of pets. Determine the mean number of pets per family.

  0, 3, 3, 3, 5, 6, and 8

  [One way to interpret the mean for this data set is to say that if the pets are redistributed evenly, each family will have 4 pets.]

- The heights of five trees in front of the school are given below. What is the average height of these trees? Does this average seem to represent the 'typical' size of these trees? Explain your answer.

  3 ft., 4 ft., 4 ft., 4 ft., 20 ft.

5.5.C Construct and interpret line graphs.

Line graphs are used to display changes in data over time.

Example:

- Below is a line graph that shows the temperature of a can of juice after the can has been placed in ice and salt over a period of time. Describe any conclusions you can make about the data.

Temperature of Apple Juice After Cooling in Salt and Ice
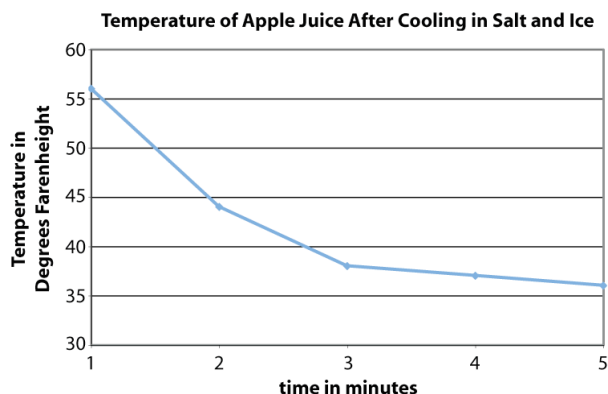
Grade 5

5.6. Core Processes: *Reasoning, problem solving, and communication*

Students in grade five solve problems that extend their understanding of core mathematical concepts—such as division of multi-digit numbers, perimeter, area, addition and subtraction of fractions and decimals, and use of variables in expressions and equations—as they make strategic decisions leading to reasonable solutions. Students use pictures, symbols, or mathematical language to explain the reasoning behind their decisions and solutions. They further develop their problem-solving skills by making generalizations about the processes used and applying these generalizations to similar problem situations. These critical reasoning, problem-solving, and communication skills represent the kind of mathematical thinking that equips students to use the mathematics they know to solve a growing range of useful and important problems and to make decisions based on quantitative information.

| *Performance Expectations*	*Explanatory Comments and Examples*
Students are expected to:	
5.6.A Determine the question(s) to be answered given a problem situation.	Descriptions of solution processes and explanations can include numbers, words (including mathematical language), pictures, physical objects, or equations. Students should be able to use all of these representations as needed. For a particular solution, students should be able to explain or show their work using at least one of these representations and verify that their answer is reasonable.
5.6.B Identify information that is given in a problem and decide whether it is essential or extraneous to the solution of the problem.	
5.6.C Determine whether additional information is needed to solve the problem.	
5.6.D Determine whether a problem to be solved is similar to previously solved problems, and identify possible strategies for solving the problem.	Examples:
5.6.E Select and use one or more appropriate strategies to solve a problem, and explain the choice of strategy.	• La Casa Restaurant uses rectangular tables. One table seats 6 people, with 1 person at each end and 2 people on each long side. However, 2 tables pushed together, short end to short end, seat only 10 people. Three tables pushed together end-to-end seat only 14 people. Write a rule that describes how many can sit at *n* tables pushed together end-to-end. The restaurant's long banquet hall has tables pushed together in a long row to seat 70. How many tables were pushed together to seat this many people? How do you know?
5.6.F Represent a problem situation using words, numbers, pictures, physical objects, or symbols.	
5.6.G Explain why a specific problem-solving strategy or procedure was used to determine a solution.	• The small square in the tangram figure below is $\frac{1}{8}$ the area of the large square.
5.6.H Analyze and evaluate whether a solution is reasonable, is mathematically correct, and answers the question.	For each of the 7 tangram pieces that make up the large square, tell what fractional part of the large square that piece represents. How do you know?
5.6.I Summarize mathematical information, draw conclusions, and explain reasoning.	

Students are expected to:

5.6.J Make and test conjectures based on data (or information) collected from explorations and experiments.
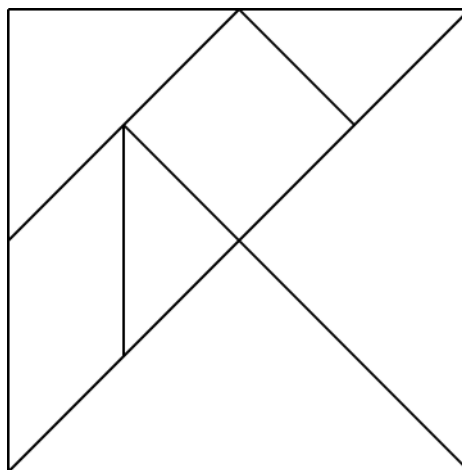
Grade 6

6.1. Core Content: *Multiplication and division of fractions and decimals (Numbers, Operations, Algebra)*

Students have done extensive work with fractions and decimals in previous grades and are now prepared to learn how to multiply and divide fractions and decimals with understanding. They can solve a wide variety of problems that involve the numbers they see every day—whole numbers, fractions, and decimals. By using approximations of fractions and decimals, students estimate computations and verify that their answers make sense.

Performance Expectations	Explanatory Comments and Examples
Students are expected to:	

6.1.A   Compare and order non-negative fractions, decimals, and integers using the number line, lists, and the symbols <, >, or =.

Examples:

- List the numbers $2\frac{1}{3}$, $\frac{4}{5}$, 0.94, $\frac{5}{4}$, 1.1, and $\frac{43}{50}$ in increasing order, and then graph the numbers on the number line.

- Compare each pair of numbers using <, >, or =.

$$\frac{4}{5} \ \square \ 1.2$$

$$\frac{7}{4} \ \square \ 1\frac{3}{4}$$

$$2\frac{7}{8} \ \square \ 2.5$$

6.1.B   Represent multiplication and division of non-negative fractions and decimals using area models and the number line, and connect each representation to the related equation.

This expectation addresses the conceptual meaning of multiplication and division of fractions and decimals. Students should be familiar with the use of visual representations like pictures (e.g., sketching the problem, grid paper) and physical objects (e.g., tangrams, cuisenaire rods). They should connect the visual representation to the corresponding equation.

The procedures for multiplying fractions and decimals are addressed in 6.1.D and 6.1.E.

6.1.C   Estimate products and quotients of fractions and decimals.

Example:

- $0.28 \div 0.96 \approx 0.3 \div 1$; $0.3 \div 1 = 0.3$

$$0.24 \times 12.4 \approx \frac{1}{4} \times 12.4 \,; \quad \frac{1}{4} \times 12.4 = 3.1$$

$$\frac{3}{13} \times \frac{20}{41} \approx \frac{1}{4} \times \frac{1}{2} \,; \quad \frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$$

6.1.D   Fluently and accurately multiply and divide non-negative fractions and explain the inverse relationship between multiplication and division with fractions.

Students should understand the inverse relationship between multiplication and division, developed in grade three and now extended to fractions. Students should work with different types of rational numbers, including whole numbers and mixed numbers, as they continue to expand their understanding of the set of rational numbers.

Examples:

- Multiply or divide.

$$\frac{4}{5} \times \frac{2}{3} \qquad\qquad 6 \div \frac{3}{8}$$

$$2\frac{1}{4} \times 3\frac{1}{2} \qquad\qquad 4\frac{1}{5} \div 1\frac{2}{3}$$

6.1.E   Multiply and divide whole numbers and decimals by 1000, 100, 10, 1, 0.1, 0.01, and 0.001.

This expectation extends what students know about the place value system and about multiplication and division and expands their set of mental math tools. As students work with multiplication by these powers of 10, they can gain an understanding of how numbers relate to each other based on their relative sizes.

Example:

- Mentally compute 0.01 x 435.

6.1.F   Fluently and accurately multiply and divide non-negative decimals.

Students should understand the inverse relationship between multiplication and division, developed in grade three and now extended to decimals. Students should work with different types of decimals, including decimals greater than 1, decimals less than 1, and whole numbers, as they continue to expand their understanding of the set of rational numbers.

Example:

- Multiply or divide.
  0.84×1.5 2.04×32
  $7.85 \div 0.32$ $17.28 \div 1.2$

6.1.G   Describe the effect of multiplying or dividing a number by one, by zero, by a number between zero and one, and by a number greater than one.

Examples:

- Without doing any computation, list
  74, 0.43×74, and $74 \div 0.85$
  in increasing order and explain your reasoning.

- Explain why $\dfrac{4}{0}$ is undefined.

Performance Expectations

Students are expected to:

6.1.H   Solve single- and multi-step word problems involving operations with fractions and decimals and verify the solutions.

Explanatory Comments and Examples

The intent of this expectation is for students to show their work, explain their thinking, and verify that the answer to the problem is reasonable in terms of the original context and the mathematics used to solve the problem. Verifications can include the use of numbers, words, pictures, or equations.

Example:

- Every day has 24 hours. Ali sleeps 3/8 of the day. Dawson sleeps 1/3 of the day. Maddie sleeps 7.2 hours in a day. Who sleeps the longest? By how much?

Grade 6

6.2. Core Content: *Mathematical expressions and equations*   *(Operations, Algebra)*

Students continue to develop their understanding of how letters are used to represent numbers in mathematics—an important foundation for algebraic thinking. Students use tables, words, numbers, graphs, and equations to describe simple linear relationships. They write and evaluate expressions and write and solve equations. By developing these algebraic skills at the middle school level, students will be able to make a smooth transition to high school mathematics.

Performance Expectations	*Explanatory Comments and Examples*
Students are expected to:	

6.2.A Write a mathematical expression or equation with variables to represent information in a table or given situation.	Examples: • What expression can be substituted for the question mark? • A t-shirt printing company charges $7 for each t-shirt it prints. Write an equation that represents the total cost, *c*, for ordering a specific quantity, *t*, of these t-shirts.

x	1	2	3	4	...	x
y	2.5	5	7.5	10	...	?

6.2.B Draw a first-quadrant graph in the coordinate plane to represent information in a table or given situation.	Example: • Mikayla and her sister are making beaded bracelets to sell at a school craft fair. They can make two bracelets every 30 minutes. Draw a graph that represents the number of bracelets the girls will have made at any point during the 6 hours they work.
6.2.C Evaluate mathematical expressions when the value for each variable is given.	Examples: • Evaluate $2s + 5t$ when $s = 3.4$ and $t = 1.8$. • Evaluate $\frac{2}{3}x - 14$ when $x = 60$.
6.2.D Apply the commutative, associative, and distributive properties, and use the order of operations to evaluate mathematical expressions.	Examples: • Simplify $6\left(\frac{1}{2} + \frac{1}{3}\right)$, with and without the use of the distributive property. • Evaluate $b - 3(2a - 7)$ when $a = 5.4$ and $b = 31.7$.

Performance Expectations
Students are expected to:

Explanatory Comments and Examples

6.2.E Solve one-step equations and verify solutions.

Students solve equations using number sense, physical objects (e.g., balance scales), pictures, or properties of equality.

Example:

- Solve for the variable in each equation below.

  $112 = 7a$

  $1.4y = 42$

  $$2\frac{1}{2} = b + \frac{1}{3}$$

  $$\frac{y}{45} = \frac{7}{15}$$

6.2.F Solve word problems using mathematical expressions and equations and verify solutions.

The intent of this expectation is for students to show their work, explain their thinking, and verify that the answer to the problem is reasonable in terms of the original context and the mathematics used to solve the problem. Verifications can include the use of numbers, words, pictures, or equations.

Example:

- Zane and his friends drove across the United States at an average speed of 55 mph. Write expressions to show how far they traveled in 12 hours, in 18 hours, and in *n* hours. How long did it take them to drive 1,430 miles? Verify your solution.

Grade 6

6.3. Core Content: *Ratios, rates, and percents*   *(Numbers, Operations, Geometry/Measurement, Algebra, Data/Statistics/Probability)*

Students extend their knowledge of fractions to develop an understanding of what a ratio is and how it relates to a rate and a percent. Fractions, ratios, rates, and percents appear daily in the media and in everyday calculations like determining the sale price at a retail store or figuring out gas mileage. Students solve a variety of problems related to such situations. A solid understanding of ratios and rates is important for work involving proportional relationships in grade seven.

Performance Expectations	**Explanatory Comments and Examples**
Students are expected to:	
6.3.A Identify and write ratios as comparisons of part-to-part and part-to-whole relationships.	Example: • If there are 10 boys and 12 girls in a class, what is the ratio of boys to girls? What is the ratio of the number of boys to the total number of students in the class?
6.3.B Write ratios to represent a variety of rates.	Example: • Julio drove his car 579 miles and used 15 gallons of gasoline. How many miles per gallon did his car get during the trip? Explain your answer.
6.3.C Represent percents visually and numerically, and convert between the fractional, decimal, and percent representations of a number.	In addition to general translations among these representations, this expectation includes the quick recall of equivalent forms of common fractions (with denominators like 2, 3, 4, 5, 8, and 10), decimals, and percents. It also includes the understanding that a fraction represents division, an important conceptual background for writing fractions as decimals. Examples: • Represent $\dfrac{75}{100}$ as a percent using numbers, a picture, and a circle graph. • Represent 40% as a fraction and as a decimal. • Write $\dfrac{13}{16}$ as a decimal and as a percent.

Performance Expectations

Students are expected to:

6.3.D Solve single- and multi-step word problems involving ratios, rates, and percents, and verify the solutions.

6.3.E Identify the ratio of the circumference to the diameter of a circle as the constant π, and recognize $\frac{22}{7}$ and 3.14 as common approximations of π.

6.3.F Determine the experimental probability of a simple event using data collected in an experiment.

6.3.G Determine the theoretical probability of an event and its complement and represent the probability as a fraction or decimal from 0 to 1 or as a percent from 0 to 100.

Explanatory Comments and Examples

The intent of this expectation is for students to show their work, explain their thinking, and verify that the answer to the problem is reasonable in terms of the original context and the mathematics used to solve the problem. Verifications can include the use of numbers, words, pictures, or equations.

Examples:

- An item is advertised as being 25% off the regular price. If the sale price is $42, what was the original regular price? Verify your solution.

- Sally had a business meeting in a city 100 miles away. In the morning, she drove an average speed of 60 miles per hour, but in the evening when she returned, she averaged only 40 miles per hour. How much longer did the evening trip take than the morning trip? Explain your reasoning.

Example:

- Measure the diameter and circumference of several circular objects. Divide each circumference by its diameter. What do you notice about the results?

The term *experimental probability* refers here to the relative frequency that was observed in an experiment.

Example:

- Tim is checking the apples in his orchard for worms. Selecting apples at random, he finds 9 apples with worms and 63 apples without worms. What is the experimental probability that a given apple from his orchard has a worm in it?

Example:

- A bag contains 4 green marbles, 6 red marbles, and 10 blue marbles. If one marble is drawn randomly from the bag, what is the probability it will be red? What is the probability that it will not be red?

Grade 6

6.4. Core Content: *Two- and three-dimensional figures*   *(Geometry/Measurement, Algebra)*

Students extend what they know about area and perimeter to more complex two-dimensional figures, including circles. They find the surface area and volume of simple three-dimensional figures. As they learn about these important concepts, students can solve problems involving more complex figures than in earlier grades and use geometry to deal with a wider range of situations. These fundamental skills of geometry and measurement are increasingly called for in the workplace and they lead to a more formal study of geometry in high school.

Performance Expectations	Explanatory Comments and Examples
Students are expected to:	

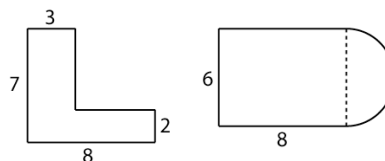|	
6.4.A Determine the circumference and area of circles.	Examples: • Determine the area of a circle with a diameter of 12 inches. • Determine the circumference of a circle with a radius of 32 centimeters.
6.4.B Determine the perimeter and area of a composite figure that can be divided into triangles, rectangles, and parts of circles.	Although students have worked with various quadrilaterals in the past, this expectation includes other quadrilaterals such as trapezoids or irregular quadrilaterals, as well as any other composite figure that can be divided into figures for which students have calculated areas before. Example: • Determine the area and perimeter of each of the following figures, assuming that the dimensions on the figures are in feet. The curved portion of the second figure is a semi-circle.
6.4.C Solve single- and multi-step word problems involving the relationships among radius, diameter, circumference, and area of circles, and verify the solutions.	The intent of this expectation is for students to show their work, explain their thinking, and verify that the answer to the problem is reasonable in terms of the original context and the mathematics used to solve the problem. Verifications can include the use of numbers, words, pictures, or equations. Example: • Captain Jenkins determined that the distance around a circular island is 44 miles. What is the distance from the shore to the buried treasure in the center of the island? What is the area of the island?

6.4.D   Recognize and draw two-dimensional representations of three-dimensional figures.

The net of a rectangular prism consists of six rectangles that can then be folded to make the prism. The net of a cylinder consists of two circles and a rectangle.

Example:

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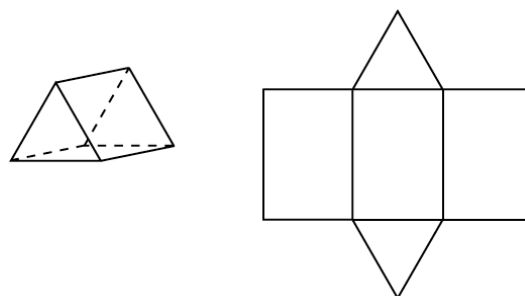
6.4.E   Determine the surface area and volume of rectangular prisms using appropriate formulas and explain why the formulas work.

Students may determine surface area by calculating the area of the faces and adding the results.

6.4.F   Determine the surface area of a pyramid.

6.4.G   Describe and sort polyhedra by their attributes: parallel faces, types of faces, number of faces, edges, and vertices.

Prisms and pyramids are the focus at this level.

Examples:

•   How many pairs of parallel faces does each polyhedron have? Explain your answer.

•   What type of polyhedron has two parallel triangular faces and three non-parallel rectangular faces?

Grade 6
6.5. Additional Key Content
(Numbers, Operations)

Students extend their mental math skills now that they have learned all of the operations—addition, subtraction, multiplication, and division—with whole numbers, fractions, and decimals. Students continue to expand their understanding of our number system as they are introduced to negative numbers for describing positions or quantities below zero. These numbers are a critical foundation for algebra, and students will learn how to add, subtract, multiply, and divide positive and negative numbers in seventh grade as further preparation for algebraic study.

Performance Expectations	Explanatory Comments and Examples
Students are expected to:	
6.5.A Use strategies for mental computations with non-negative whole numbers, fractions, and decimals.	Examples:

- John wants to find the total number of hours he worked this week. Use his time card below to find the total.

Days	Monday	Tuesday	Wednesday	Thursday	Friday
Days	$4\frac{1}{4}$	3	$6\frac{1}{2}$	$7\frac{1}{2}$	$1\frac{1}{2}$

- What is the total cost for items priced at $25.99 and $32.95? (A student may think of something like 25.99 + 32.95 = (26 + 33) − 0.06 = 58.94.)

6.5.B Locate positive and negative integers on the number line and use integers to represent quantities in various contexts.

Contexts could include elevation, temperature, or debt, among others.

6.5.C Compare and order positive and negative integers using the number line, lists, and the symbols <, >, or =.

Examples:

- Compare each pair of numbers using <, >, or =.

−11 □ −14

−7 □ 4

−101 □ −94

Grade 6

6.6. Core Processes: Reasoning, problem solving, and communication

Students refine their reasoning and problem-solving skills as they move more fully into the symbolic world of algebra and higher-level mathematics. They move easily among representations—numbers, words, pictures, or symbols—to understand and communicate mathematical ideas, to make generalizations, to draw logical conclusions, and to verify the reasonableness of solutions to problems. In grade six, students solve problems that involve fractions and decimals as well as rates and ratios in preparation for studying proportional relationships and algebraic reasoning in grade seven.

Performance Expectations	*Explanatory Comments and Examples*
Students are expected to:	

6.6.A Analyze a problem situation to determine the question(s) to be answered.

6.6.B Identify relevant, missing, and extraneous information related to the solution to a problem.

6.6.C Analyze and compare mathematical strategies for solving problems, and select and use one or more strategies to solve a problem.

6.6.D Represent a problem situation, describe the process used to solve the problem, and verify the reasonableness of the solution.

6.6.E Communicate the answer(s) to the question(s) in a problem using appropriate representations, including symbols and informal and formal mathematical language.

6.6.F Apply a previously used problem-solving strategy in a new context.

6.6.G Extract and organize mathematical information from symbols, diagrams, and graphs to make inferences, draw conclusions, and justify reasoning.

6.6.H Make and test conjectures based on data (or information) collected from explorations and experiments.

Descriptions of solution processes and explanations can include numbers, words (including mathematical language), pictures, physical objects, or equations. Students should be able to use all of these representations as needed. For a particular solution, students should be able to explain or show their work using at least one of these representations and verify that their answer is reasonable.

Examples:

- As part of her exercise routine, Carmen jogs twice around the perimeter of a square park that measures $\frac{5}{8}$ mile on each side. On Monday, she started at one corner of the park and jogged $\frac{2}{3}$ of the way around in 17 minutes before stopping at a small pond in the park to feed some ducks. How far had Carmen run when she reached the pond? What percent of her planned total distance had Carmen completed when she stopped to feed the ducks? If it took Carmen 17 minutes to jog to the point where she stopped, assuming that she continued running in the same direction at the same pace and did not stop again, how long would it have taken her to get back to her starting point? Explain your answers.

- At Springhill Elementary School's annual fair, Vanessa is playing a game called "Find the Key." A key is randomly placed somewhere in one of the rooms shown on the map below. (The key cannot be placed in the hallway.)

To win the game, Vanessa must correctly guess the room where the key is placed. Use what you know about the sizes of the rooms to determine the probability that the key is placed in the gym, the office, the café, the book closet, or the library. Write each probability as a simplified fraction, a decimal, and a percent. Which room should Vanessa select in order to have the best chance of winning? Justify the solution.
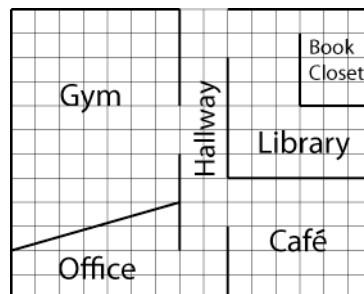
Grade 7

7.1. Core Content: *Rational numbers and linear equations*   *(Numbers, Operations, Algebra)*

Students add, subtract, multiply, and divide rational numbers—fractions, decimals, and integers—including both positive and negative numbers. With the inclusion of negative numbers, students can move more deeply into algebraic content that involves the full set of rational numbers. They also approach problems that deal with a wider range of contexts than before. Using generalized algebraic skills and approaches, students can approach a wide range of problems involving any type of rational number, adapting strategies for solving one problem to different problems in different settings with underlying similarities.

Performance Expectations	*Explanatory Comments and Examples*
Students are expected to:	
7.1.A Compare and order rational numbers using the number line, lists, and the symbols <, >, or =.	Examples:
	• List the numbers $\frac{2}{3}, -\frac{2}{3}, 1.2, \frac{4}{3}, -\frac{4}{3}, -1.2$, and $-\frac{7}{4}$ in increasing order, and graph the numbers on the number line.
	• Compare each pair of numbers using <, >, or =.
	$$\frac{-11}{20} \ \square \ \frac{-13}{21}$$ $$\frac{-7}{5} \ \square \ -1.35$$ $$-2\frac{3}{4} \ \square \ -2.75$$
7.1.B Represent addition, subtraction, multiplication, and division of positive and negative integers visually and numerically.	Students should be familiar with the use of the number line and physical materials, such as colored chips, to represent computation with integers. They should connect numerical and physical representations to the computation. The procedures are addressed in 7.1.C.
	Examples:
	• Use a picture, words, or physical objects to illustrate $3 - 7$; $-3 - 7$; $-3 - (-7)$; $(-3)(-7)$; $21 \div (-3)$.
	• At noon on a certain day, the temperature was $13°$; at 10 p.m. the same day, the temperature was $-8°$. How many degrees did the temperature drop between noon and 10 p.m.?

Performance Expectations

Students are expected to:

Explanatory Comments and Examples

7.1.C Fluently and accurately add, subtract, multiply, and divide rational numbers.

This expectation brings together what students know about the four operations with positive and negative numbers of all kinds—integers, fractions, and decimals. Some of these skills will have been recently learned and may need careful development and reinforcement.

This is an opportunity to demonstrate connections among the operations and to show similarities and differences in the performance of these operations with different types of numbers. Visual representations may be helpful as students begin this work, and they may become less necessary as students become increasingly fluent with the operations.

Examples:

- $-\dfrac{4}{3} - \dfrac{3}{4} =$

- $\dfrac{-272}{8} =$

- $(3.5)(-6.4) =$

7.1.D Define and determine the absolute value of a number.

Students define absolute value as the distance of the number from zero.

Examples:

- Explain why 5 and –5 have the same absolute value.

- Evaluate $|7.8 - 10.3|$.

7.1.E Solve two-step linear equations.

Example:

- Solve $3.5x - 12 = 408$ and show each step in the process.

7.1.F Write an equation that corresponds to a given problem situation, and describe a problem situation that corresponds to a given equation.

Students have represented various types of problems with expressions and particular types of equations in previous grades. Many students at this grade level will also be able to deal with inequalities.

Examples:

- Meagan spent $56.50 on 3 blouses and a pair of jeans. If each blouse cost the same amount and the jeans cost $25, write an algebraic equation that represents this situation and helps you determine how much one blouse cost.

- Describe a problem situation that could be solved using the equation $15 = 2x - 7$.

7.1.G   Solve single- and multi-step word problems involving rational numbers and verify the solutions.

The intent of this expectation is for students to show their work, explain their thinking, and verify that the answer to the problem is reasonable in terms of the original context and the mathematics used to solve the problem. Verifications can include the use of numbers, words, pictures, or equations.

Example:

- Tom wants to buy some candy bars and magazines for a trip. He has decided to buy three times as many candy bars as magazines. Each candy bar costs $0.70 and each magazine costs $2.50. The sales tax rate on both types of items is $6\frac{1}{2}\%$. How many of each item can he buy if he has $20.00 to spend?

7.2. Core Content: *Proportionality and similarity*   (*Operations, Geometry/Measurement, Algebra*)

Students extend their work with ratios to solve problems involving a variety of proportional relationships, such as making conversions between measurement units or finding the percent increase or decrease of an amount. They also solve problems involving the proportional relationships found in similar figures, and in so doing reinforce an important connection between numerical operations and geometric relationships. Students graph proportional relationships and identify the rate of change as the slope of the related line. The skills and concepts related to proportionality represent some of the most important connecting ideas across K–12 mathematics. With a good understanding of how things grow proportionally, students can understand the linear relationships that are the basis for much of high school mathematics. If learned well, proportionality can open the door for success in much of secondary mathematics.

Performance Expectations	*Explanatory Comments and Examples*
Students are expected to:	
7.2.A Mentally add, subtract, multiply, and divide simple fractions, decimals, and percents.	Example: • A shirt is on sale for 20% off the original price of $15. Use mental math strategies to calculate the sale price of the shirt.
7.2.B Solve single- and multi-step problems involving proportional relationships and verify the solutions.	The intent of this expectation is for students to show their work, explain their thinking, and verify that the answer to the problem is reasonable in terms of the original context and the mathematics used to solve the problem. Verifications can include the use of numbers, words, pictures, or equations. Problems include those that involve rate, percent increase or decrease, discount, markup, profit, interest, tax, or the conversion of money or measurement (including multiplying or dividing amounts in recipes). More complex problems, such as dividing 100 into more than two proportional parts (e.g., 4:3:3), allow students to generalize what they know about proportional relationships to a range of situations. Examples: • At a certain store, 48 television sets were sold in April. The manager at the store wants to encourage the sales team to sell more TVs and is going to give all the sales team members a bonus if the number of TVs sold increases by 30% in May. How many TVs must the sales team sell in May to receive the bonus? Explain your answer.

Performance Expectations		**Explanatory Comments and Examples**
Students are expected to:		

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• After eating at a restaurant, you know that the bill before tax is $52.60 and that the sales tax rate is 8%. You decide to leave a 20% tip for the waiter based on the pre-tax amount. How much should you leave for the waiter? How much will the total bill be, including tax and tip? Show work to support your answers.

• Joe, Sam, and Jim completed different amounts of yard work around the school. They agree to split the $200 they earned in a ratio of 5:3:2, respectively. How much did each boy receive?
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7.2.C Describe proportional relationships in similar figures and solve problems involving similar figures.

Students should recognize the constant ratios in similar figures and be able to describe the role of a scale factor in situations involving similar figures. They should be able to connect this work with more general notions of proportionality.

Example:

• The length of the shadow of a tree is 68 feet at the same time that the length of the shadow of a 6-foot vertical pole is 8 feet. What is the height of the tree?

7.2.D Make scale drawings and solve problems related to scale.

Example:

• On an 80:1 scale drawing of the floor plan of a house, the dimensions of the living room are $1\frac{7}{8}'' \times 2\frac{1}{2}''$. What is the actual area of the living room in square feet?

7.2.E Represent proportional relationships using graphs, tables, and equations, and make connections among the representations.

Proportional relationships are linear relationships whose graphs pass through the origin and can be written in the form $y = kx$.

Example:

• The relationship between the width and length of similar rectangles is shown in the table below. Write an equation that expresses the length, *l*, in terms of the width, *w*, and graph the relationship between the two variables.

width	4	12	18	...	w
length	10	30	45	...	?

Performance Expectations	Explanatory Comments and Examples
Students are expected to:	

7.2.F Determine the slope of a line corresponding to the graph of a proportional relationship and relate slope to similar triangles.

This expectation connects the constant rate of change in a proportional relationship to the concept of slope of a line. Students should know that the slope of a line is the same everywhere on the line and realize that similar triangles can be used to demonstrate this fact. They should recognize how proportionality is reflected in slope as it is with similar triangles. A more complete discussion of slope is developed in high school.

7.2.G Determine the unit rate in a proportional relationship and relate it to the slope of the associated line.

The associated unit rate, constant rate of change of the function, and slope of the graph all represent the constant of proportionality in a proportional relationship.

Example:

- Coffee costs $18.96 for 3 pounds. What is the cost per pound of coffee? Draw a graph of the proportional relationship between the number of pounds of coffee and the total cost, and describe how the unit rate is represented on the graph.

7.2.H Determine whether or not a relationship is proportional and explain your reasoning.

A proportional relationship is one in which two quantities are related by a constant scale factor, k. It can be written in the form $y = kx$. A proportional relationship has a constant rate of change and its graph passes through the origin.

Example:

- Determine whether each situation represents a proportional relationship and explain your reasoning.

x	1	2	3	4
y	4.5	9	13.5	18

  — $y = 3x + 2$

  — One way to calculate a person's maximum target heart rate during exercise in beats per minute is to subtract the person's age from 200. Is the relationship between the maximum target heart rate and age proportional? Explain your reasoning.

Students are expected to:

7.2.I Solve single- and multi-step problems involving conversions within or between measurement systems and verify the solutions.	The intent of this expectation is for students to show their work, explain their thinking, and verify that the answer to the problem is reasonable in terms of the original context and the mathematics used to solve the problem. Verifications can include the use of numbers, words, pictures, or equations.

Students should be given the conversion factor when converting between measurement systems.

Examples:

- The lot that Dana is buying for her new one-story house is 35 yards by 50 yards. Dana's house plans show that her house will cover 1,600 square feet of land. What percent of Dana's lot will not be covered by the house? Explain your work.

- Joe was planning a business trip to Canada, so he went to the bank to exchange $200 U.S. dollars for Canadian dollars (at a rate of $1.02 CDN per $1 US). On the way home from the bank, Joe's boss called to say that the destination of the trip had changed to Mexico City. Joe went back to the bank to exchange his Canadian dollars for Mexican pesos (at a rate of 10.8 pesos per $1 CDN). How many Mexican pesos did Joe get?

Grade 7

7.3. Core Content: *Surface area and volume*   *(Geometry/Measurement)*

Students extend their understanding of surface area and volume to include finding surface area and volume of cylinders and volume of cones and pyramids. They apply formulas and solve a range of problems involving three-dimensional objects, including problems people encounter in everyday life, in certain types of work, and in other school subjects. With a strong understanding of how to work with both two-dimensional and three-dimensional figures, students build an important foundation for the geometry they will study in high school.

Performance Expectations	*Explanatory Comments and Examples*
Students are expected to:	

7.3.A Determine the surface area and volume of cylinders using the appropriate formulas and explain why the formulas work.	Explanations might include the use of models such as physical objects or drawings. A net can be used to illustrate the formula for finding the surface area of a cylinder.
7.3.B Determine the volume of pyramids and cones using formulas.	
7.3.C Describe the effect that a change in scale factor on one attribute of a two- or three-dimensional figure has on other attributes of the figure, such as the side or edge length, perimeter, area, surface area, or volume of a geometric figure.	Examples: • A cube has a side length of 2 cm. If each side length is tripled, what happens to the surface area? What happens to the volume? • What happens to the area of a circle if the diameter is decreased by a factor of 3?
7.3.D Solve single- and multi-step word problems involving surface area or volume and verify the solutions.	The intent of this expectation is for students to show their work, explain their thinking, and verify that the answer to the problem is reasonable in terms of the original context and the mathematics used to solve the problem. Verifications can include the use of numbers, words, pictures, or equations. Examples: • Alexis needs to paint the four exterior walls of a large rectangular barn. The length of the barn is 80 feet, the width is 50 feet, and the height is 30 feet. The paint costs $28 per gallon, and each gallon covers 420 square feet. How much will it cost Alexis to paint the barn? Explain your work. • Tyesha has decided to build a solid concrete pyramid on her empty lot. The base will be a square that is forty feet by forty feet and the height will be thirty feet. The concrete that she will use to construct the pyramid costs $70 per cubic yard. How much will the concrete for the pyramid cost Tyesha? Justify your answer.

Grade 7

7.4. Core Content: *Probability and data* *(Data/Statistics/Probability)*

Students apply their understanding of rational numbers and proportionality to concepts of probability. They begin to understand how probability is determined, and they make related predictions. Students revisit how to interpret data, now using more sophisticated types of data graphs and thinking about the meaning of certain statistical measures. Statistics, including probability, is considered one of the most important and practical fields of study for making sense of quantitative information, and it plays an important part in secondary mathematics in the 21st century.

Performance Expectations	Explanatory Comments and Examples
Students are expected to:	
7.4.A Represent the sample space of probability experiments in multiple ways, including tree diagrams and organized lists.	The sample space is the set of all possible outcomes. Example: • José flips a penny, Jane flips a nickel, and Janice flips a dime, all at the same time. List the possible outcomes of the three simultaneous coin flips using a tree diagram or organized list.
7.4.B Determine the theoretical probability of a particular event and use theoretical probability to predict experimental outcomes.	Example: • A triangle with a base of 8 units and a height of 7 units is drawn inside a rectangle with an area of 90 square units. What is the probability that a randomly selected point inside the rectangle will also be inside the triangle? • There are 5 blue, 4 green, 8 red, and 3 yellow marbles in a paper bag. Rosa runs an experiment in which she draws a marble from the bag, notes the color on a sheet of paper, and puts the marble back in the bag, repeating the process 200 times. About how many times would you expect Rosa to draw a red marble?
7.4.C Describe a data set using measures of center (median, mean, and mode) and variability (maximum, minimum, and range) and evaluate the suitability and limitations of using each measure for different situations.	As a way to understand these ideas, students could construct data sets for a given mean, median, mode, or range. Examples: • Kiley keeps track of the money she spends each week for two months and records the following amounts: $6.30, $2.25, $43.00, $2.25, $11.75, $5.25, $4.00, and $5.20. Which measure of center is most representative of Kiley's weekly spending? Support your answer. • Construct a data set with five data points, a mean of 24, a range of 10, and without a mode.

- A group of seven adults have an average age of 36. If the ages of three of the adults are 45, 30, and 42, determine possible ages for the remaining four adults.

7.4.D   Construct and interpret histograms, stem-and-leaf plots, and circle graphs.

7.4.E   Evaluate different displays of the same data for effectiveness and bias, and explain reasoning.

Example:

- The following two bar graphs of the same data show the number of five different types of sodas that were sold at Blake High School. Compare and contrast the two graphs. Describe a reason why you might choose to use one graph over the other.
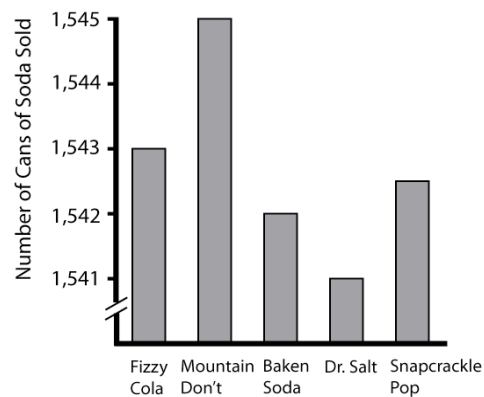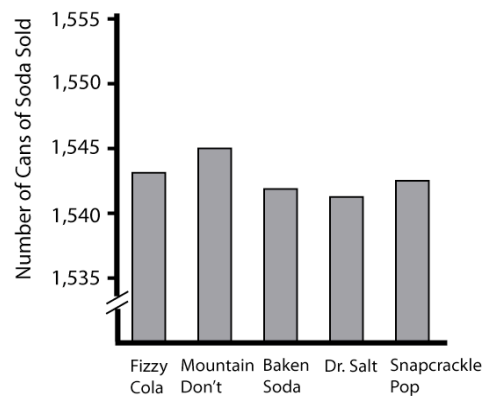

Figure 1

Figure 2

Grade 7
7.5. Additional Key Content
(Numbers, Algebra)

Students extend their coordinate graphing skills to plotting points with both positive and negative coordinates on the coordinate plane. Using pairs of numbers to locate points is a necessary skill for reading maps and tables and a critical foundation for high school mathematics. Students further prepare for algebra by learning how to use exponents to write numbers in terms of their most basic (prime) factors.

Performance Expectations	*Explanatory Comments and Examples*
Students are expected to:	
7.5.A Graph ordered pairs of rational numbers and determine the coordinates of a given point in the coordinate plane.	Example: • Graph and label the points A(1, 2), B(–1, 5), C(–3, 2), and D(–1, –5). Connect the points in the order listed and identify the figure formed by the four points. • Graph and label the points A(1, –2), B(–4, –2), and C(–4, 3). Determine the coordinates of the fourth point (D) that will complete the figure to form a square. Graph and label point D on the coordinate plane and draw the resulting square.
7.5.B Write the prime factorization of whole numbers greater than 1, using exponents when appropriate.	Writing numbers in prime factorization is a useful tool for determining the greatest common factor and least common multiple of two or more numbers. Example: • Write the prime factorization of 360 using exponents.

Grade 7

7.6. Core Processes: *Reasoning, problem solving, and communication*

Students refine their reasoning and problem-solving skills as they move more fully into the symbolic world of algebra and higher-level mathematics. They move easily among representations—numbers, words, pictures, or symbols—to understand and communicate mathematical ideas, to make generalizations, to draw logical conclusions, and to verify the reasonableness of solutions to problems. In grade seven, students solve problems that involve positive and negative numbers and often involve proportional relationships. As students solve these types of problems, they build a strong foundation for the study of linear functions that will come in grade eight.

Performance Expectations	**Explanatory Comments and Examples**
Students are expected to:	
7.6.A Analyze a problem situation to determine the question(s) to be answered.	Descriptions of solution processes and explanations can include numbers, words (including mathematical language), pictures, physical objects, or equations. Students should be able to use all of these representations as needed. For a particular solution, students should be able to explain or show their work using at least one of these representations and verify that their answer is reasonable.
7.6.B Identify relevant, missing, and extraneous information related to the solution to a problem.	
7.6.C Analyze and compare mathematical strategies for solving problems, and select and use one or more strategies to solve a problem.	
7.6.D Represent a problem situation, describe the process used to solve the problem, and verify the reasonableness of the solution.	Examples:
	• When working on a report for class, Catrina read that a person over the age of 30 can lose approximately 0.06 centimeters of height per year. Catrina's 80-year-old grandfather is 5 feet 7 inches tall. Assuming her grandfather's height has decreased at this rate, about how tall was he at age 30? Catrina's cousin, Richard, is 30 years old and is 6 feet 3 inches tall. Assuming his height also decreases approximately 0.06 centimeters per year after the age of 30, about how tall will you expect him to be at age 55? (Remember that 1 inch ≈ 2.54 centimeters.) Justify your solution.
7.6.E Communicate the answer(s) to the question(s) in a problem using appropriate representations, including symbols and informal and formal mathematical language.	
7.6.F Apply a previously used problem-solving strategy in a new context.	
7.6.G Extract and organize mathematical information from symbols, diagrams, and graphs to make inferences, draw conclusions, and justify reasoning.	• If one man takes 1.5 hours to dig a 5-ft × 5-ft × 3-ft hole, how long will it take three men working at the same pace to dig a 10-ft × 12-ft × 3-ft hole? Explain your solution.
7.6.H Make and test conjectures based on data (or information) collected from explorations and experiments.	

Grade 8
8.1. Core Content: *Linear functions and equations*   *(Algebra)*

S tudents solve a variety of linear equations and inequalities. They build on their familiarity with proportional relationships and simple linear equations to work with a broader set of linear relationships, and they learn what functions are. They model applied problems with mathematical functions represented by graphs and other algebraic techniques. This Core Content area includes topics typically addressed in a high school algebra or a first-year integrated math course, but here this content is expected of all middle school students in preparation for a rich high school mathematics program that goes well beyond these basic algebraic ideas.

Performance Expectations	*Explanatory Comments and Examples*
Students are expected to:	

8.1.A   Solve one-variable linear equations.

Examples:

Solve each equation for *x*.

- $91 - 2.5x = 26$

- $\dfrac{7}{8}(x - 2) = 119$

- $-3x + 34 = 5x$

- $114 = -2x - 8 + 5x$

- $3(x - 2) - 4x = 2(x + 22) - 5$

8.1.B   Solve one- and two-step linear inequalities and graph the solutions on the number line.

The emphasis at this grade level is on gaining experience with inequalities, rather than on becoming proficient at solving inequalities in which multiplying or dividing by a negative is necessary.

Example:

- Graph the solution of $4x - 21 > 57$ on the number line.

8.1.C   Represent a linear function with a verbal description, table, graph, or symbolic expression, and make connections among these representations.

Translating among these various representations of functions is an important way to demonstrate a conceptual understanding of functions.

8.1.D   Determine the slope and *y*-intercept of a linear function described by a symbolic expression, table, or graph.

Examples:

- Determine the slope and *y*-intercept for the function described by

$$y = \frac{2}{3}x - 5$$

- The following table represents a linear function. Determine the slope and *y*-intercept.

x	2	3	5	8	12
y	5	8	14	23	35

Students are expected to:

8.1.E Interpret the slope and *y*-intercept of the graph of a linear function representing a contextual situation.

Example:

- A car is traveling down a long, steep hill. The elevation, *E*, above sea level (in feet) of the car when it is *d* miles from the top of the hill is given by $E = 7500 – 250d$, where *d* can be any number from 0 to 6. Find the slope and *y*-intercept of the graph of this function and explain what they mean in the context of the moving car.

8.1.F Solve single- and multi-step word problems involving linear functions and verify the solutions.

The intent of this expectation is for students to show their work, explain their thinking, and verify that the answer to the problem is reasonable in terms of the original context and the mathematics used to solve the problem. Verifications can include the use of numbers, words, pictures, or equations.

Example:

- Mike and Tim leave their houses at the same time to walk to school. Mike's walk can be represented by $d_1 = 4000 – 400t$, and Tim's walk can be represented by $d_2 = 3400 – 250t$, where *d* is the distance from the school in feet and *t* is the walking time in minutes. Who arrives at school first? By how many minutes? Is there a time when Mike and Tim are the same distance away from the school? Explain your reasoning.

8.1.G Determine and justify whether a given verbal description, table, graph, or symbolic expression represents a linear relationship.

Examples:

- Could the data presented in the table represent a linear function? Explain your reasoning.

x	-1	0	1	2	3	4	5
y	0	-1	0	3	8	15	24

- Does $y = \frac{1}{4}x - 5$ represent a linear function? Explain your reasoning.

Grade 8
8.2. Core Content: *Properties of geometric figures*   *(Numbers, Geometry/Measurement)*

Students work with lines and angles, especially as they solve problems involving triangles. They use known relationships involving sides and angles of triangles to find unknown measures, connecting geometry and measurement in practical ways that will be useful well after high school. Since squares of numbers arise when using the Pythagorean Theorem, students work with squares and square roots, especially in problems with two- and three-dimensional figures. Using basic geometric theorems such as the Pythagorean Theorem, students get a preview of how geometric theorems are developed and applied in more formal settings, which they will further study in high school.

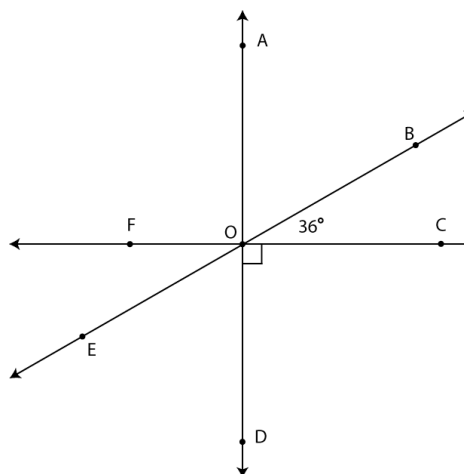| *Performance Expectations*	*Explanatory Comments and Examples*
Students are expected to:	
8.2.A Identify pairs of angles as complementary, supplementary, adjacent, or vertical, and use these relationships to determine missing angle measures.	Example: • Determine the measures of ∠BOA, ∠EOD, ∠FOB, and ∠FOE and explain how you found each measure. As part of your explanation, identify pairs of angles as complementary, supplementary, or vertical.
8.2.B Determine missing angle measures using the relationships among the angles formed by parallel lines and transversals.	Example: • Determine the measures of the indicated angles. ∠1: _____ ∠2: _____ ∠3: _____ ∠4: _____

Performance Expectations	Explanatory Comments and Examples
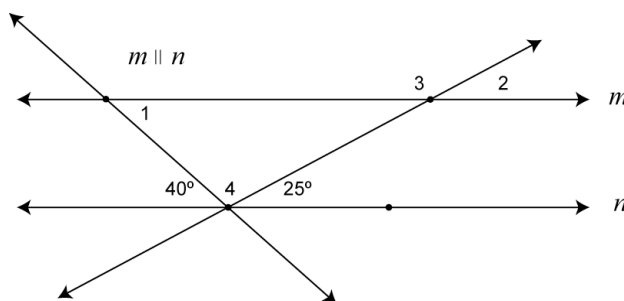
Students are expected to:

8.2.C Demonstrate that the sum of the angle measures in a triangle is 180 degrees, and apply this fact to determine the sum of the angle measures of polygons and to determine unknown angle measures.

Examples:

- Determine the measure of each interior angle in a regular pentagon.

- In a certain triangle, the measure of one angle is four times the measure of the smallest angle, and the measure of the remaining angle is the sum of the measures of the other two angles. Determine the measure of each angle.

8.2.D Represent and explain the effect of one or more translations, rotations, reflections, or dilations (centered at the origin) of a geometric figure on the coordinate plane.

Example:

- Consider a trapezoid with vertices (1,2), (1,6), (6,4), and (6,2). The trapezoid is reflected across the *x*-axis and then translated four units to the left. Graph the image of the trapezoid after these two transformations and give the coordinates of the new vertices.

8.2.E Quickly recall the square roots of the perfect squares from 1 through 225 and estimate the square roots of other positive numbers.

Students can use perfect squares of integers to determine squares and square roots of related numbers, such as 1.96 and 0.0049.

Examples:

- Determine: $\sqrt{36}$, $\sqrt{0.25}$, $\sqrt{144}$, and $\sqrt{196}$.

- Between which two consecutive integers does the square root of 74 lie?

8.2.F Demonstrate the Pythagorean Theorem and its converse and apply them to solve problems.

One possible demonstration is to start with a right triangle, use each of the three triangle sides to form the side of a square, and draw the remaining three sides of each of the three squares. The areas of the three squares represent the Pythagorean relationship.

Examples:

- Is a triangle with side lengths 5 cm, 12 cm, and 13 cm a right triangle? Why or why not?

- Determine the length of the diagonal of a rectangle that is 7 ft by 10 ft.

8.2.G Apply the Pythagorean Theorem to determine the distance between two points on the coordinate plane.

Example:

- Determine the distance between the points (–2, 3) and (4, 7).

Grade 8

8.3. Core Content: *Summary and analysis of data sets*   *(Algebra, Data/Statistics/Probability)*

Students build on their extensive experience organizing and interpreting data and apply statistical principles to analyze statistical studies or short statistical statements, such as those they might encounter in newspapers, on television, or on the Internet. They use mean, median, and mode to summarize and describe information, even when these measures may not be whole numbers. Students use their knowledge of linear functions to analyze trends in displays of data. They create displays for two sets of data in order to compare the two sets and draw conclusions. They expand their work with probability to deal with more complex situations than they have previously seen. These concepts of statistics and probability are important not only in students' lives, but also throughout the high school mathematics program.

Performance Expectations	Explanatory Comments and Examples
Students are expected to:	
8.3.A Summarize and compare data sets in terms of variability and measures of center.	Students use mean, median, mode, range, and interquartile range to summarize and compare data sets, and explain the influence of outliers on each measure.
	Example:
	• Captain Bob owns two charter boats, the *Sock-Eye-To-Me* and *Old Gus*, which take tourists on fishing trips. On Saturday, the *Sock-Eye-To-Me* took four people fishing and returned with eight fish weighing 18, 23, 20, 6, 20, 22, 18, and 20 pounds. On the same day, *Old Gus* took five people fishing and returned with ten fish weighing 38, 18, 12, 14, 17, 42, 12, 16, 12, and 14 pounds.
	Using measures of center and variability, compare the weights of the fish caught by the people in the two boats.
	Make a summary statement telling which boat you would charter for fishing based on thse data and why.
	What influence, if any, do outliers have on the particular statistics for these data?
8.3.B Select, construct, and analyze data displays, including box-and-whisker plots, to compare two sets of data.	Previously studied displays include stem-and-leaf plots, histograms, circle graphs, and line plots. Here these displays are used to compare data sets. Box-and-whisker plots are introduced here for the first time as a powerful tool for comparing two or more data sets.
	Example:
	• As part of their band class, Tayla and Alyssa are required to keep practice records that show the number of minutes they practice their instruments each day. Below are their

Tayla: 55, 45, 60, 45, 30, 30, 90, 50, 40, 75, 25, 90, 105, 60

Alyssa: 20, 120, 25, 20, 0, 15, 30, 15, 90, 0, 30, 30, 10, 30

Of stem-and-leaf plot, circle graph, or line plot, select the data display that you think will best compare the two girls' practice records. Construct a display to show the data. Compare the amount of time the two girls practice by analyzing the data presented in the display.

8.3.C	Create a scatterplot for a two-variable data set, and, when appropriate, sketch and use a trend line to make predictions.

Example:

- Kera randomly selected seventeen students from her middle school for a study comparing arm span to standing height. The students' measurements are shown in the table below.

Comparison of Arm Span and Standing Height (in cm) at Icicle River Middle School

Height (cm)	Arm Span (cm)	Height (cm)	Arm Span (cm)
138	145	155	150
135	135	175	177
142	147	162	160
158	145	150	152
177	174	142	143
150	152	149	149
158	162	160	165
160	160	173	170
160	158		

Create a scatterplot for the data shown.

If appropriate, sketch a trendline.

Use these data to estimate the arm span of a student with a height of 180 cm, and the height of a student with an arm span of 130 cm. Explain any limitations of using this process to make estimates.

Performance Expectations

Students are expected to:

8.3.D Describe different methods of selecting statistical samples and analyze the strengths and weaknesses of each method.

8.3.E Determine whether conclusions of statistical studies reported in the media are reasonable.

8.3.F Determine probabilities for mutually exclusive, dependent, and independent events for small sample spaces.

Explanatory Comments and Examples

Students should work with a variety of sampling techniques and should be able to identify strengths and weakness of random, census, convenience, and representative sampling.

Example:

- Carli, Jamar, and Amberly are conducting a survey to determine their school's favorite Seattle professional sports team. Carli selects her sample using a convenience method—she surveys students on her bus during the ride to school. Jamar uses a computer to randomly select 30 numbers from 1 through 600, and then surveys the corresponding students from a numbered, alphabetical listing of the student body. Amberly waits at the front entrance before school and surveys every twentieth student entering. Analyze the strengths and weaknesses of each method.

Examples:

- Given a standard deck of 52 playing cards, what is the probability of drawing a king or queen? [Some students may be unfamiliar with playing cards, so alternate examples may be desirable.]

- Leyanne is playing a game at a birthday party. Beneath ten paper cups, a total of five pieces of candy are hidden, with one piece hidden beneath each of five cups. Given only three guesses, Leyanne must uncover three pieces of candy to win all the hidden candy. What is the probability she will win all the candy?

- A bag contains 7 red marbles, 5 blue marbles, and 8 green marbles. If one marble is drawn at random and put back in the bag, and then a second marble is drawn at random, what is the probability of drawing first a red marble, then a blue marble?

Students are expected to:

8.3.G   Solve single- and multi-step problems using counting techniques and Venn diagrams and verify the solutions**.**

The intent of this expectation is for students to show their work, explain their thinking, and verify that the answer to the problem is reasonable in terms of the original context and the mathematics used to solve the problem. Verifications can include the use of numbers, words, pictures, or equations.

Counting techniques include the fundamental counting principle, lists, tables, tree diagrams, etc.

Examples:

- Jack's Deli makes sandwiches that include a choice of one type of bread, one type of cheese, and one type of meat. How many different sandwiches could be made given 4 different bread types, 3 different cheeses, and 5 different meats? Explain your reasoning.

- A small high school has 57 tenth-graders. Of these students, 28 are taking geometry, 34 are taking biology, and 10 are taking neither geometry nor biology. How many students are taking both geometry and biology? How many students are taking geometry but not biology? How many students are taking biology but not geometry? Draw a Venn diagram to illustrate this situation.

Grade 8
8.4. Additional Key Content

(Numbers, Operations)

Students deal with a few key topics about numbers as they prepare to shift to higher level mathematics in high school. First, they use scientific notation to represent very large and very small numbers, especially as these numbers are used in technological fields and in everyday tools like calculators or personal computers. Scientific notation has become especially important as "extreme units" continue to be identified to represent increasingly tiny or immense measures arising in technological fields. A second important numerical skill involves using exponents in expressions containing both numbers and variables. Developing this skill extends students' work with order of operations to include more complicated expressions they might encounter in high school mathematics. Finally, to help students understand the full breadth of the real-number system, students are introduced to simple irrational numbers, thus preparing them to study higher level mathematics in which properties and procedures are generalized for the entire set of real numbers.

Performance Expectations	Explanatory Comments and Examples
Students are expected to:	
8.4.A Represent numbers in scientific notation, and translate numbers written in scientific notation into standard form.	Examples: • Represent 4.27×10^{-3} in standard form. • Represent 18,300,000 in scientific notation. • Throughout the year 2004, people in the city of Cantonville sent an average of 400 million text messages a day. Using this information, about how many total text messages did Cantonville residents send in 2004? (2004 was a leap year.) Express your answer in scientific notation.
8.4.B Solve problems involving operations with numbers in scientific notation and verify solutions.	Units include those associated with technology, such as nanoseconds, gigahertz, kilobytes, teraflops, etc. Examples: • A supercomputer used by a government agency will be upgraded to perform 256 teraflops (that is, 256 trillion calculations per second). Before the upgrade, the supercomputer performs 1.6×10^{13} calculations per second. How many more calculations per second will the upgraded supercomputer be able to perform? Express the answer in scientific notation. • A nanosecond is one billionth of a second. How many nanoseconds are there in five minutes? Express the answer in scientific notation.

Performance Expectations	Explanatory Comments and Examples
Students are expected to:	

8.4.C Evaluate numerical expressions involving non-negative integer exponents using the laws of exponents and the order of operations.

Example:

- Simplify and write the answer in exponential form:

$$\frac{(7^4)^2}{7^3}$$

Some students will be ready to solve problems involving simple negative exponents and should be given the opportunity to do so.

Example:

- Simplify and write the answer in exponential form:

$$(5^4)^2 5^{-3}$$

8.4.D Identify rational and irrational numbers.

Students should know that rational numbers are numbers that can be represented as the ratio of two integers; that the decimal expansions of rational numbers have repeating patterns, or *terminate*; and that there are numbers that are not rational.

Example:

- Identify whether each number is rational or irrational and explain your choice.

$$3.14, \quad 4.\overline{6}, \quad \frac{1}{11}, \quad \sqrt{2}, \quad \sqrt{25}, \quad \pi$$

Grade 8

8.5. Core Processes: *Reasoning, problem solving, and communication*

Students refine their reasoning and problem-solving skills as they move more fully into the symbolic world of algebra and higher level mathematics. They move easily among representations—numbers, words, pictures, or symbols—to understand and communicate mathematical ideas, to make generalizations, to draw logical conclusions, and to verify the reasonableness of solutions to problems. In grade eight, students solve problems that involve proportional relationships and linear relationships, including applications found in many contexts outside of school. These problems dealing with proportionality continue to be important in many applied contexts, and they lead directly to the study of algebra. Students also begin to deal with informal proofs for theorems that will be proven more formally in high school.

Performance Expectations	Explanatory Comments and Examples
Students are expected to:	
8.5.A Analyze a problem situation to determine the question(s) to be answered.	Descriptions of solution processes and explanations can include numbers, words (including mathematical language), pictures, or equations. Students should be able to use all of these representations as needed. For a particular solution, students should be able to explain or show their work using at least one of these representations and verify that their answer is reasonable.
8.5.B Identify relevant, missing, and extraneous information related to the solution to a problem.	
8.5.C Analyze and compare mathematical strategies for solving problems, and select and use one or more strategies to solve a problem.	
8.5.D Represent a problem situation, describe the process used to solve the problem, and verify the reasonableness of the solution.	Examples:
	• The dimensions of a room are 12 feet by 15 feet by 10 feet. What is the furthest distance between any two points in the room? Explain your solution.
8.5.E Communicate the answer(s) to the question(s) in a problem using appropriate representations, including symbols and informal and formal mathematical language.	• Miranda's phone service contract ends this month. She is looking for ways to save money and is considering changing phone companies. Her current cell phone carrier, X-Cell, calculates the monthly bill using the equation $c = \$15.00 + \$0.07m$, where c represents the total monthly cost and m represents the number of minutes of talk time during a monthly billing cycle. Another company, Prism Cell, offers 300 free minutes of talk time each month for a base fee of $30.00 with an additional $0.15 for every minute over 300 minutes. Miranda's last five phone bills were $34.95, $36.70, $37.82, $62.18, and $36.28. Using the data from the last five months, help Miranda decide whether she should switch companies. Justify your answer.
8.5.F Apply a previously used problem-solving strategy in a new context.	
8.5.G Extract and organize mathematical information from symbols, diagrams, and graphs to make inferences, draw conclusions, and justify reasoning.	
8.5.H Make and test conjectures based on data (or information) collected from explorations and experiments.	